NOTES

- NOTES WITH MIND MAPS -

MATHEMATICS

(ARITHMETIC PROGRESSIONS)

10TH

Arithmetic Progressions

1. What is a Sequence?

- A **sequence** is an arrangement of numbers in a definite order according to some rule.
- The various numbers occurring in a sequence are called its **terms**.
- We denote the terms of a sequence by a_1, a_2, a_3…etc. Here, the subscripts denote the positionsof the terms in the sequence.
- In general, the number at the n^{th} place is called the n^{th} term of the sequence and is denoted by an. The n^{th} term is also called the **general term** of the sequence.
- A sequence having a finite number of terms is called a **finite sequence**.
- A sequence which do not have a last term and which extends indefinitely is known as an **infinite sequence**.

Sequences, Series and Progressions

A sequence is a finite or infinite list of numbers following a specific pattern. For example, 1, 2, 3, 4, 5,… is the sequence, an infinite sequence of natural numbers.

A series is the sum of the elements in the corresponding sequence. For example, 1 + 2 + 3 + 4 + 5….is the series of natural numbers. Each number in a sequence or a series is called a term.

A progression is a sequence in which the general term can be can be expressed using a mathematical formula.

2. Arithmetic Progression:

- An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term, except the first term.
- Each of the numbers of the sequence is called a **term** of an Arithmetic Progression. The fixed number is called the **common difference**. This common difference could be a positive number, a negative number or even zero.

3. General form and general term (n^{th} term) of an A.P:

- The **general form of an A.P.** is a, a + d, a + 2d, a + 3d….., where ‘a’ is the first term and ‘d’ is the common difference.
- The **general term (n^{th} term)** of an A.P is given by $a_n = a + (n - 1) d$, where ‘a’ is the first term and ‘d’ is the common difference.
- If the A.P a, a + d, a + 2d, …….., l is reversed to l, l – d, l – 2d, ….., a then the common difference changes to negative of the common difference of the original sequence.
- To find the **n^{th} term from the end**, we consider this AP backward such that the last term becomes the first term.

l, (l - d), (l - 2d) ……

The general term of this AP is given by $a_n = \ell + (n - 1)(-d)$

4. Algorithm to determine whether a sequence is an AP or not:

When we are given an algebraic formula for the general term of the sequence:

Step 1: Obtain a_n.

Step 2: Replace n by (n + 1) in an to get a_{n+1}

Step 3: Calculate $a_{n+1} - a_n$

Step 4: Check the value of $a_{n+1} - a_n$.

If $a_{n+1} - a_n$ is independent of n, then the given sequence is an A.P. Otherwise, it is not an A.P.

OR

A list of numbers a_1, a_2, a_3…… is an A.P, if the differences $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$ … give the same value, i.e., $a_{k+1} - a_k$ is same for all different values of k.

5. Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

It should be noted that in case of an odd number of terms, the middle term is ‘a’ and the common difference is ‘d’ while in case of an even number of terms the middle terms are a – d, a + d and the common differences is 2d.

6. Arithmetic mean:

If three number a, b, c (in order) are in A.P. Then,

$b - a = c - b$ = common difference

$\Rightarrow 2b = a + c$

Thus a, b and c are in A.P., if and only if 2b = a + c. In this case, b is called the **Arithmetic mean** of a and c.

7. Sum of n terms of an A.P:

➢ **Sum of n terms of an A.P.** is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where 'a' is the first term, 'd' is the common difference and 'n' is the total number of terms.

- **Sum of n terms of an A.P.** is also given by:

$$S_n = \frac{n}{2}[a + l]$$

where 'a' is the first term and 'λ' is the last term.

- **Sum of first n natural numbers** is given by $\frac{n(n+1)}{2}$

8. The n^{th} term of an A.P is the difference of the sum to first n terms and the sum to first (n - 1) terms of it. That is, $a_n = S_n - S_{n-1}$

9. Common Difference

The difference between two consecutive terms in an AP, (which is constant) is the "common difference"(d) of an A.P. In the progression: 2, 5, 8, 11, 14 …the common difference is 3.

As it is the difference between any two consecutive terms, for any A.P, if the common difference is:

- positive, the AP is increasing.
- zero, the AP is constant.
- negative, the A.P is decreasing.

10.Finite and Infinite AP

- A finite AP is an A.P in which the number of terms is finite. For example the A.P: 2, 5, 8……32, 35, 38
- An infinite A.P is an A.P in which the number of terms is infinite. For example: 2, 5, 8, 11…..

A finite A.P will have the last term, whereas an infinite A.P won't.

Finite sets are the sets having a finite/countable number of members. Finite sets are also known as countable sets as they can be counted. The process will run out of elements to list if the elements of this set have a finite number of members.

Examples of finite sets:

P = {0, 3, 6, 9, …, 99}

Q = {a : a is an integer, 1 < a < 10}

A set of all English Alphabets (because it is countable).

Another example of a Finite set:

A set of months in a year.

M = {January, February, March, April, May, June, July, August, September, October, November, December}

n (M) = 12

It is a finite set because the number of elements is countable.

Cardinality of Finite Set

If 'a' represents the number of elements of set A, then the cardinality of a finite set is n(A) = a.

So, the Cardinality of the set A of all English Alphabets is 26, because the number of elements (alphabets) is 26.

Hence, n (A) = 26.

Similarly, for a set containing the months in a year will have a cardinality of 12.

So, this way we can list all the elements of any finite set and list them in the curly braces or in Roster form.

Properties of Finite sets

The following finite set conditions are always finite.

- A subset of Finite set
- The union of two finite sets
- The power set of a finite set

Few Examples:

P = {1, 2, 3, 4}

Q = {2, 4, 6, 8}

R = {2, 3)

Here, all the P, Q, R are the finite sets because the elements are finite and countable.

R

⊂

P, i.e R is a Subset of P because all the elements of set R are present in P. So, the subset of a finite set is always finite.

P U Q is {1, 2, 3, 4, 6, 8}, so the union of two sets is also finite.

The number of elements of a power set = 2^n.

The number of elements of the power set of set P is 24 = 16, as the number of elements of set P is 4. So it shows that the power set of a finite set is finite.

Non- Empty Finite set

It is a set where either the number of elements are big or only starting or ending is given. So, we denote it with the number of elements with n(A) and if n(A)is a natural number then it's a finite set.

Example:

S = {a set of the number of people living in India}

It is difficult to calculate the number of people living in India but it's somewhere a natural number. So, we can call it a non-empty finite set.

If N is a set of natural numbers less than n. So the cardinality of set N is n.

N = {1,2,3….n}

$X = x_1, x_2, \ldots\ldots, x_n$

$Y = \{x : x_1 \in N, 1 \leq i \leq n\}$, where i is the integer between 1 and n.

Can we say that an empty set is a finite set?

Let's learn what is an empty set first.

An empty set is a set which has no elements in it and can be represented as { } and shows that it has no element.

P = { } Or ∅

As the finite set has a countable number of elements and the empty set has zero elements so, it is a definite number of elements.

So, with a cardinality of zero, an empty set is a finite set.

What is Infinite set?

If a set is not finite, it is called an infinite set because the number of elements in that set is not countable and also we cannot represent it in Roster form. Thus, infinite sets are also known as uncountable sets.

So, the elements of an Infinite set are represented by 3 dots (ellipse) thus, it represents the infinity of that set.

Examples of Infinite Sets

- A set of all whole numbers, W = {0, 1, 2, 3, 4,…}
- A set of all points on a line
- The set of all integers

Cardinality of Infinite Sets

The cardinality of a set is n (A) = x, where x is the number of elements of a set A. The cardinality of an infinite set is n (A) = ∞ as the number of elements is unlimited in it.

Properties of Infinite Sets

- The union of two infinite sets is infinite
- The power set of an infinite set is infinite
- The superset of an infinite set is also infinite

11.Comparison of Finite and Infinite Sets

Let's compare the differences between Finite and Infinite set:

The sets could be equal only if their elements are the same, so a set could be equal only if it is a finite set, whereas if the elements are not comparable, the set is infinite.

Factors	Finite sets	Infinite sets
Number of elements	Elements are countable	The number of elements is uncountable
Continuity	It has a start and end elements	It is endless from the start or end. Both the sides could have continuity
Cardinality	n(A) = n, n is the number of elements in the set	n(A) = ∞ as the number of elements are uncountable
union	Union of two finite	Union of two infinite sets is infinite

Factors	Finite sets	Infinite sets
	sets is finite	
Power set	The power set of a finite set is also finite	The power set of an infinite set is infinite
Roster form	Can be easily represented in roster form	As the set in infinite set can't be represented in Roster form, so we use three dots to represent the infinity

How to know if a Set is Finite or Infinite?

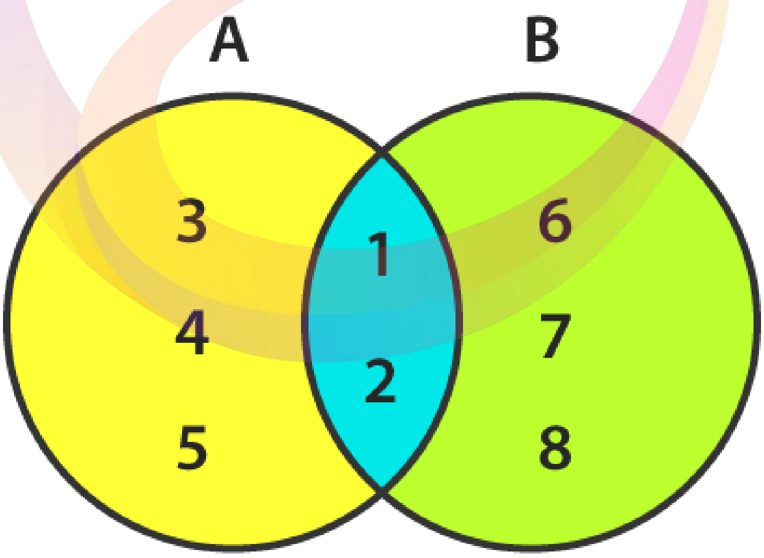
As we know that if a set has a starting point and an ending point both, it is a finite set, but it is infinite if it has no end from any side or both sides.

Points to identify a set is whether a finite or infinite are:

An infinite set is endless from the start or end, but both the side could have continuity unlike in Finite set where both start and end elements are there.

If a set has the unlimited number of elements, then it is infinite and if the elements are countable then it is finite.

12.Graphical Representation of Finite and Infinite Sets



Here in the above picture,

A = {1, 2, 3, 4, 5}

B = {1, 2, 6, 7, 8}

A U B = {1, 2, 3, 4, 5, 6, 7, 8}

A∩B = {1, 2}

Both A and B are finite sets as they have a limited number of elements.

n(A) = 5 and n(B) = 5

AUB and A∩B are also finite.

So, a Venn diagram can represent the finite set but it is difficult to do the same for an infinite set as the number of elements can't be counted and bounced in a circle