NOTES

- NOTES WITH MIND MAPS -

MATHEMATICS

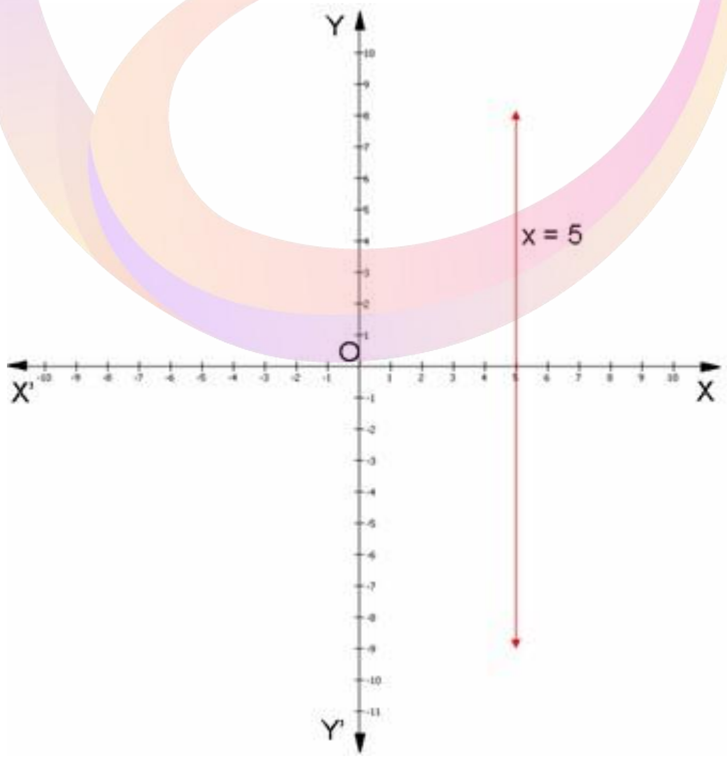
(LINEAR EQUATIONS IN TWO VARIABLES)

9TH

Linear Equations in Two Variables

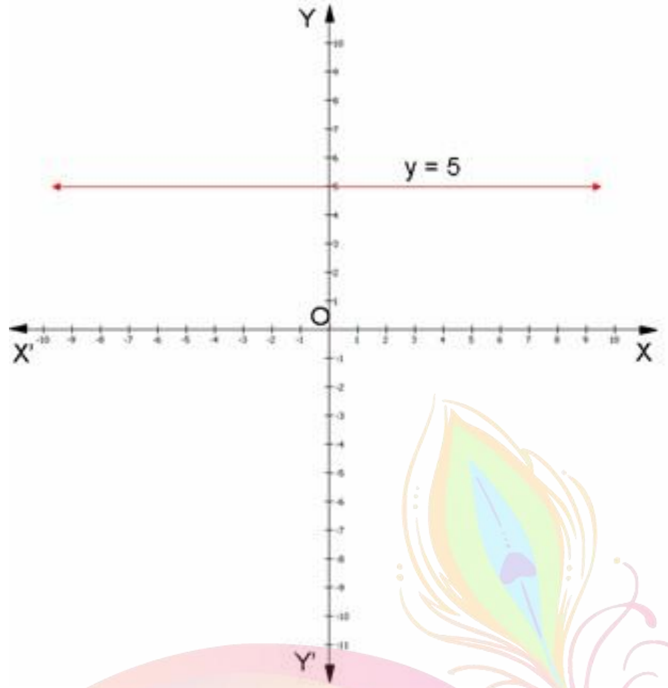
1. An equation of the form ax + by + c = 0, where a, b and c are real numbers, such that a and b are not both zero, is called a **linear equation in two variables**.
2. Linear equations in one variable, of the type ax + b = 0, can also expressed as a linear equation in two variables. Since, ax + b = 0 ⇒ ax + 0.y + b =0.
3. A **solution** of a linear equation in two variables is a pair of values, one for x and one for y, which satisfy the equation.
4. The solution of a linear equation is not affected when-
   i. The same number is added or subtracted from both the sides of an equation.
   ii. Multiplying or dividing both the sides of the equation by the same non-zero number.
5. A linear equation in two variables has **infinitely many solutions**.
6. Every point on the line satisfies the equation of the line and every solution of the equation is a point on the line.
7. A linear equation in two variables is represented geometrically by a straight line whose points make up the collection of solutions of the equation. This is called the **graph** of the linear equation.
8. x = 0 is the equation of the y-axis and y = 0 is the equation of the x-axis.
9. The graph of **x = k** is a straight line parallel to the y-axis.

   For example, the graph of the equation x = 5 is as follows:

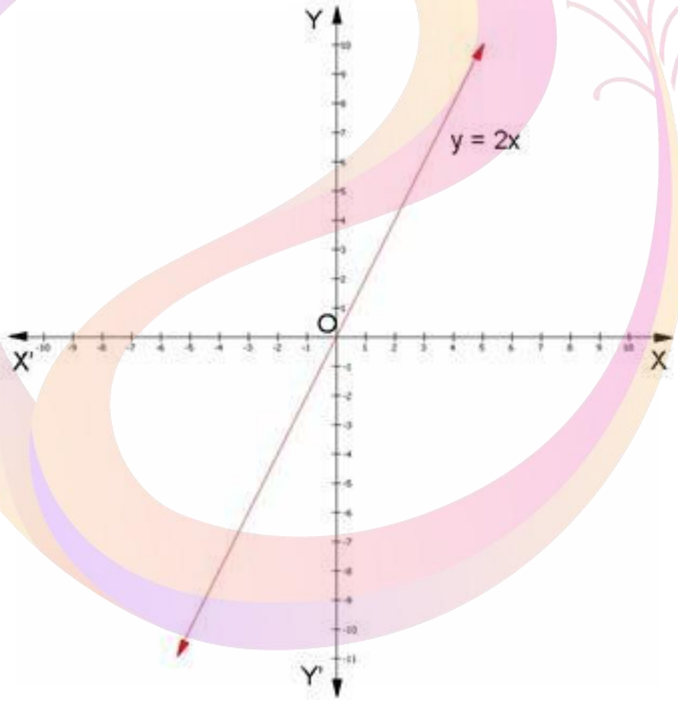


10. The graph of **y = k** is a straight line parallel to the x-axis.

For example, the graph of the equation y = 5 is as follows:



11. An equation of the type **y = mx** represents a line passing through the origin, where m is a real number. For example, the graph of the equation y = 2x is as follows:



Linear equation in one variable

When an equation has only one variable of degree one, then that equation is known as linear equation in one variable.

A linear equation in one variable is an equation which has a maximum of one variable of order 1. It is of the form ax + b = 0, where x is the variable.

- Standard form: ax + b = 0, where a and b ϵ R & a ≠ 0
- Examples of linear equation in one variable are:

– 3x - 9 = 0

– 2t = 5

Standard Form of Linear Equations in One Variable

The standard form of linear equations in one variable is represented as:

ax + b = 0

Where,

- 'a' and 'b' are real numbers.
- Both 'a' and 'b' are not equal to zero.

Thus, the formula of linear equation in one variable is ax + b = 0.

Solving Linear Equations in One Variable

For solving an equation having only one variable, the following steps are followed

- Step 1: Using LCM, clear the fractions if any.
- Step 2: Simplify both sides of the equation.
- Step 3: Isolate the variable.
- Step 4: Verify your answer.

Example of Solution of Linear Equation in One Variable

Let us understand the concept with the help of an example.

For solving equations with variables on both sides, the following steps are followed:

Consider the equation: 5x – 9 = -3x + 19

Step 1: Transpose all the variables on one side of the equation. By transpose, we mean to shift the variables from one side of the equation to the other side of the equation. In the method of transposition, the operation on the operand gets reversed.

In the equation 5x – 9 = -3x + 19, we transpose -3x from the right-hand side to the left-hand side of the equality, the operation gets reversed upon transposition and the equation becomes:

5x – 9 +3x = 19

⇒ 8x -9 = 19

Step 2: Similarly transpose all the constant terms on the other side of the equation as below:

8x -9 = 19

⇒ 8x = 19 + 9

⇒ 8x = 28

Step 3: Divide the equation with 8 on both sides of the equality.

8x/8 = 28/8

⇒ $x = 28/8$

If we substitute $x = 28/8$ in the equation $5x - 9 = -3x + 19$, we will get $9 = 9$, thereby satisfying the equality and giving us the required solution.

The Application of Linear equation

There are various applications of linear equations in Mathematics as well as in real life. An algebraic equation is an equality that includes variables and equal sign (=). A linear equation is an equation of degree one.

The knowledge of mathematics is frequently applied through word problems, and the applications of linear equations are observed on a wide scale to solve such word problems. Here, we are going to discuss the linear equation applications and how to use them in the real world with the help of an example.

A linear equation is an algebraic expression with a variable and equality sign (=), and whose highest degree is equal to 1. For example, $2x - 1 = 5$ is a linear equation.

- Linear equation with one variable and degree one is called a linear equation in one variable. (Eg, $3x + 5 = 0$)
- Linear equation with degree one and two variables is called a linear equation in two variables. (Eg, $3x + 5y = 0$)

The graphical representation of linear equation is $ax + by + c = 0$, where,

- a and b are coefficients
- x and y are variables
- c is a constant term

In real life, the applications of linear equations are vast. To tackle real-life problems using algebra, we convert the given situation into mathematical statements in such a way that it clearly illustrates the relationship between the unknowns (variables) and the information provided. The following steps are involved while restating a situation into a mathematical statement:

- Translate the problem statement into a mathematical statement and set it up in the form of algebraic expression in a manner it illustrates the problem aptly.
- Identify the unknowns in the problem and assign variables (quantity whose value can change depending upon the mathematical context) to these unknown quantities.
- Read the problem thoroughly multiple times and cite the data, phrases and keywords. Organize the information obtained sequentially.
- Frame an equation with the help of the algebraic expression and the data provided in the problem statement and solve it using systematic techniques of equation solving.
- Retrace your solution to the problem statement and analyze if it suits the criterion of the problem.

There you go!! Using these steps and applications of linear equations word problems can be solved easily.

Applications of Linear equations in Real life

- Finding unknown age
- Finding unknown angles in geometry
- For calculation of speed, distance or time
- Problems based on force and pressure

Let us look into an example to analyze the applications of linear equations in depth.

Applications of Linear Equations Solved Example

Example:

Rishi is twice as old as Vani. 10 years ago his age was thrice of Vani. Find their present ages.

Solution:

In this word problem, the ages of Rishi and Vani are unknown quantities. Therefore, as discussed above, let us first choose variables for the unknowns.

Let us assume that Vani's present age is 'x' years. Since Rishi's present age is 2 times that of Vani, therefore his present age can be assumed to be '2x'.

10 years ago, Vani's age would have been 'x – 10 ', and Rishi's age would have been '2x – 10'. According to the problem statement, 10 years ago, Rishi's age was thrice of Vani, i.e. 2x – 10 = 3(x – 10).

We have our linear equation in the variable 'x' which clearly defines the problem statement. Now we can solve this linear equation easily and get the result.

$$2x - 10 = 3(x - 10)$$
$$\Rightarrow 2x - 10 = 3x - 30$$
$$\Rightarrow x = 20$$

This implies that the current age of Vani is 20 years, and Rishi's age is '2x,' i.e. 40 years. Let us retrace our solution. If the present age of Vani is 20 years then 10 years ago her age would have been 10 years, and Rishi's age would have been 30 years which satisfies our problem statement. Thus, applications of linear equations enable us to tackle such real-world problems.

Linear Equations Formula

A linear equation looks like any other equation. It is made up of two expressions set equal to each other. It is equal to the product that is directly proportional to the other plus the constant.

The Linear equation formula is given by

$$y = mx + b$$

Where,

m determines the slope of that line,

b determines the point at which the line crosses

Solved Examples

Question 1: Solve for x: $5x + 6 = 11$

Solution:

Given function is $5x + 6 = 11$

$5x = 11 - 6$

$x = \frac{5}{5} = 1$

Therefore, $x = 1$.

Graphing of Linear Equations

Linear equations, also known as first-order degree equations, where the highest power of the variable is one. When an equation has one variable, it is known as linear equations in one variable. If the linear equations contain two variables, then it is known as linear equations in two variables, and so on. In this article, we are going to discuss the linear equations in two variables, and also going to learn about the graphing of linear equations in two variables with examples.

Linear Equations in Two Variables

Equations of degree one and having two variables are known as linear equations in two variables. It is of the form, $ax + by + c = 0$, where a, b and c are real numbers, and both a and b not equal to zero.

Equations of the form $ax + by = 0$; where a and b are real numbers, and $a, b \neq 0$, is also linear equations in two variable.

Solution of a Linear Equation in Two Variables

The solution of a linear equation in two variables is a pair of numbers, one for x and one for y which satisfies the equation. There are infinitely many solutions for a linear equation in two variables.

For example, $x + 2y = 6$ is a linear equation and some of its solution are (0,3),(6,0),(2,2) because, they satisfy $x + 2y = 6$.

Graphing of Linear Equation in Two Variables

Since the solution of linear equation in two variable is a pair of numbers (x,y), we can represent the solutions in a coordinate plane.

Consider the equation,

$2x + y = 6$ —(1)

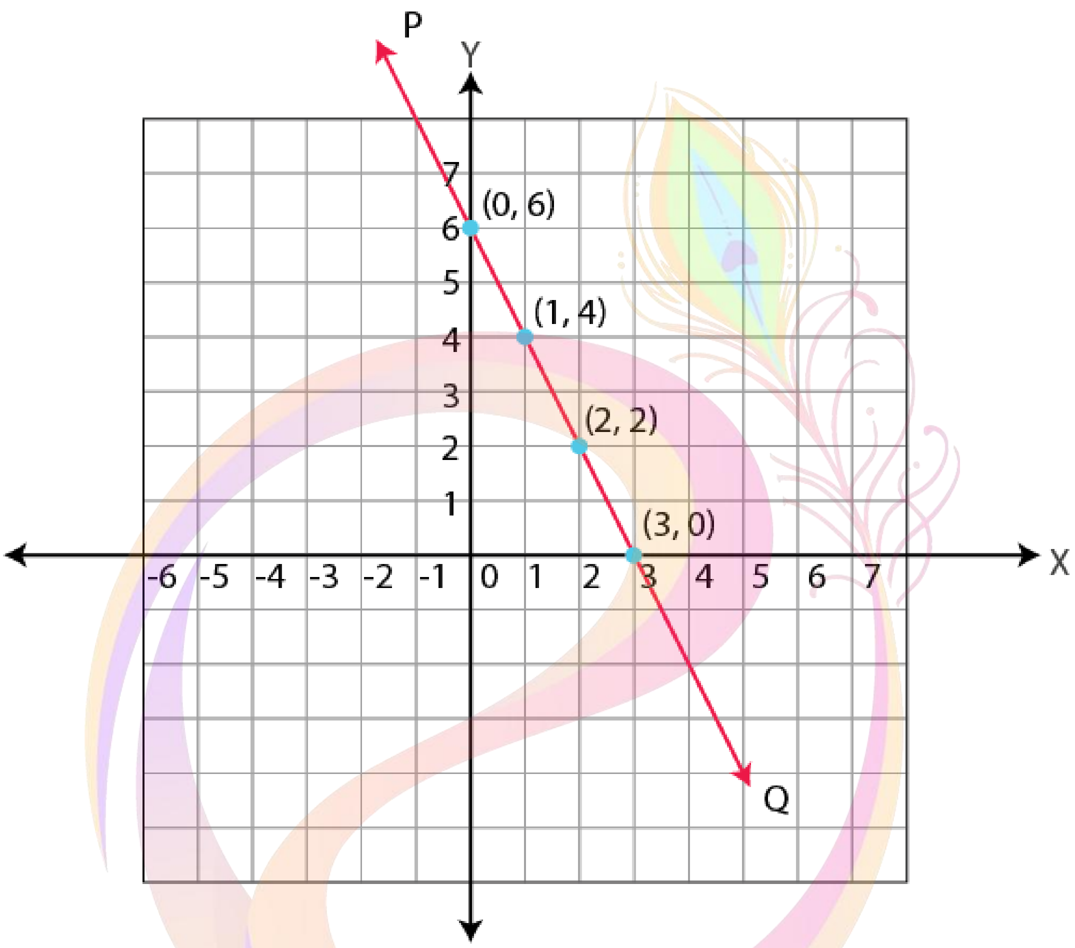
Some solutions of the above equation are, (0,6), (3,0), (1,4), (2,2) because, they satisfy (1).

We can represent the solution of (1) using a table as shown below.

x	0	3	1	2	...
y	6	0	4	2	...

We can plot the above points (0, 6), (3, 0), (1, 4), (2, 2) in a coordinate plane (Refer figure).

We can take any two points and join those to make a line. Let the line be PQ. It is observed that all the four points are lying on the same line PQ.



Consider any other point on the line PQ, for example, take point (4, -2) which lies on PQ.

Let's check whether this point satisfies the equation or not.

Substituting (4, -2) in (1) gives,

LHS = $(2 \times 4) - 2 = 6$ = RHS

Therefore (4, -2) is a solution of (1).

Similarly, if we take any point on the line PQ, it will satisfy (1).

It can be observed that,

- All the points say, (p, q) on the line PQ gives a solution of $2x + y = 6$.
- All the solution of $2x + y = 6$, lie on the line PQ.
- Points which are not the solution of $2x + y = 6$ will not lie on the line PQ.