

NOTES

- NOTES WITH MIND MAPS -
MATHEMATICS
(MENSURATION)



Mensuration

Area of polygons

- Area of a rectangle = Length×Breadth
 - Area of a square =(Side)²
 - Area of a triangle = $\frac{1}{2}$ × Base× Height
 - Area of a parallelogram = Base× Height
 - Area of a trapezium = $\frac{1}{2}$ sum of parallel sides× distance between them
 - Area of a rhombus = $\frac{1}{2}$ ×product of its diagonals
 - Area of a general quadrilateral can be found by dividing it into two triangles, by drawing on of its diagonals, and then applying the formula of area of a triangle.
1. Area of a polygon (or field) can be calculated by suitably dividing it into triangle, rectangle, trapezium etc.
 2. Surface area of a solid is the sum of the areas of all its faces.
 3. Amount of region occupied by a solid is called its volume.
 4. For a cuboid of length l, breadth b and height h, we have:
 - Volume of cuboid = (l × b × h) cubic units
 - Total surface area of cuboid = (lb + bh + lh) sq units
 - Lateral surface area of cuboid = {2(l + b) × h} sq units
 - Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$ units
 5. For a cube of side a, we have:
 - Volume of cube = (a³) cubic units
 - Total surface area of cube = (6a²) sq units
 - Lateral surface area of cube = (4a²) sq units
 - Diagonal of cube = $\sqrt{3}a$ units
 6. For a cylinder of height h and base radius r, we have:
 - Volume of cylinder = (πr²h) cubic units

- Curved surface area of cylinder = $(2\pi rh)$ squnits
- Total surface area of cylinder = $2\pi r(h + r)$ squnits
- Unit conversion:
 - $1 \text{ cm}^3 = 1 \text{ mL}$
 - $1 \text{ L} = 1000\text{cm}^3$
 - $1 \text{ m}^3 = 10^6 \text{ cm}^3 = 1000\text{L}$

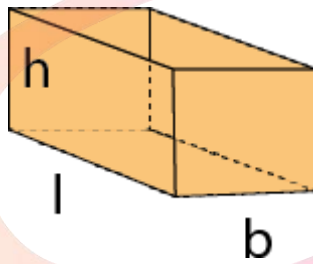
Mensuration

Mensuration is a branch of mathematics which mainly deals with the study of different kinds of Geometrical shapes along with its area, length, volume and perimeters. It is completely based on the application of both algebraic equations and geometric calculations. The results obtained by the Mensuration are considered very accurate. There are two types of geometric shapes:

Volume of a 3D Object

Volume is the space occupied by the three-dimensional object. It is a three-dimensional quantity.

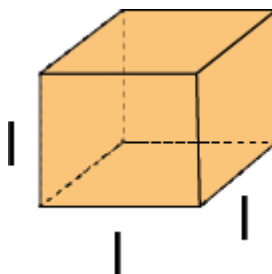
Volume of a Cuboid



$$\text{Volume of a cuboid} = l \times b \times h$$

where, l is the length, b is the breadth and h is the height of the cuboid.

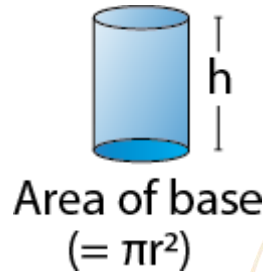
Volume of a Cube



Volume of a cube = l^3

Where, l is the length of each side of the cube.

Volume of a Cylinder



Volume of the cylinder = $\pi r^2 h$

Where r is the radius of the base and h is the height of the cylinder.

Basics Revisited

Mensuration is the study of geometry that deals with the measurement of length, areas and volumes.

Perimeter is the total length or path of a given shape.

Area is the total region covered by the given shape.

Volume is the total space occupied by the given shape.

Identifying Shapes and Areas of Different Regular Figures

Area of a Rectangle: length \times breadth, perimeter: $2(\text{length} + \text{breadth})$

Area of Square: side \times side, perimeter: $4 \times \text{side}$

Area of Triangle: $\frac{1}{2}(\text{base} \times \text{height})$, perimeter: $a + b + c$ (sum of 3 sides)

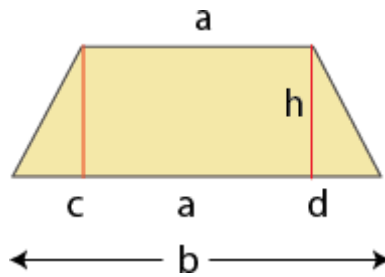
Area of Parallelogram: base \times height, perimeter: $2(\text{length} + \text{breadth})$

Area of Circle: $\pi \times (\text{radius})^2$, perimeter: $2 \times \pi \times \text{radius}$.

Trapezium

Area of Trapezium by Division into Shapes of Known Area

Consider the trapezium where a and b are parallel sides, h is the height. Trapezium is divided into 3 parts: two triangles, one rectangle.



Here h is the height, a and b are 2 parallel sides.

Area of trapezium = Area of 2 triangles + Area of rectangle

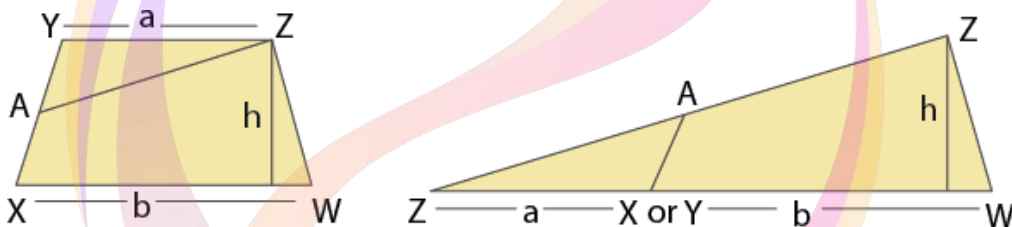
$$= \left[\frac{1}{2} \times c \times h + \frac{1}{2} \times d \times h \right] + [a \times h]$$

Area of Trapezium by Finding the Area of a Triangle of Same Area

The area of the trapezium can be found out by dividing it into a triangle and a polygon.

Consider a trapezium WXYZ. Mark a midpoint A for side XY and join AZ. Cut the trapezium along AZ and obtain a ΔAZY

Flip the ΔAZY and place it as shown below. Now the new polygon is a triangle.



We know that,

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

Substituting the values we get,

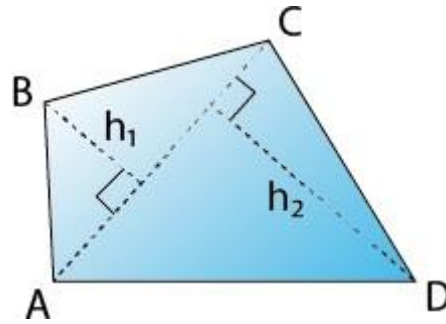
$$\text{Area of a triangle} = \frac{1}{2} \times (a+b) \times h$$

But the original polygon is a trapezium. So,

$$\text{Area of a trapezium} = \frac{1}{2} \times (a+b) \times h$$

Area of a General Quadrilateral

Consider a quadrilateral ABCD. Draw diagonal AC. From B and D draw perpendiculars h_1, h_2 to AC



Area of quadrilateral = Area of triangle ABC + Area of triangle ADC

$$= \frac{1}{2} \times \text{base} \times \text{height} + \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \left(\frac{1}{2} \times AC \times h_1\right) + \left(\frac{1}{2} \times AC \times h_2\right) \text{ [Where, } h_1, h_2 \text{ are the heights, AC is the base]}$$

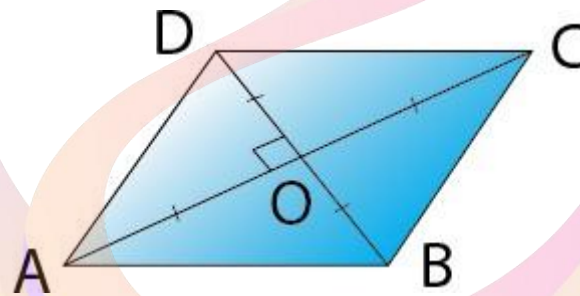
$$= \frac{1}{2} \times AC \times (h_1 + h_2) = \frac{1}{2} \times d \times (h_1 + h_2) \text{ [}\because \text{ AC is a diagonal]}$$

$$\therefore \text{Area of a Quadrilateral} = \frac{1}{2} \times d \times (h_1 + h_2)$$

where d is diagonal and h_1, h_2 are perpendicular drawn to a diagonal.

Area of Rhombus

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2,$$

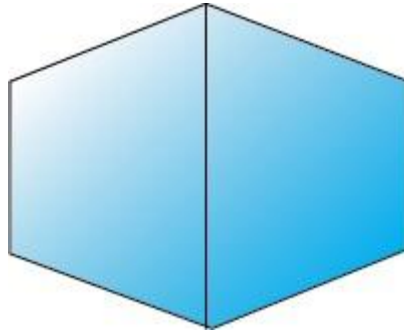


where d_1 and d_2 are the diagonals.

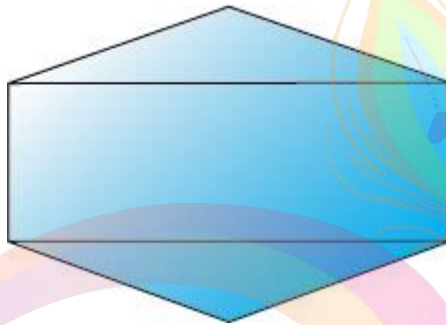
Area of Polygons

The area of any given polygon can be found by cutting the polygon into shapes whose area is known and adding the area of these shapes.

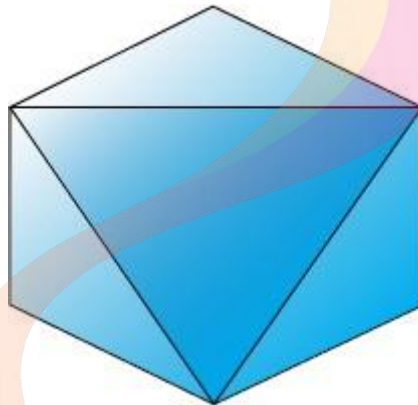
Some of the ways to find the area is shown below.



Area of this polygon = area of 2 trapeziums



Area of this polygon = Area of 2 triangles + Area of rectangle.



Area of this polygon = Area of 4 triangles.

Surface Area of Solids

Solid Shapes

Solid shapes or solid figures are the three-dimensional figures which have length, breadth and height. Using these, surface areas and volumes of these figures are found out.

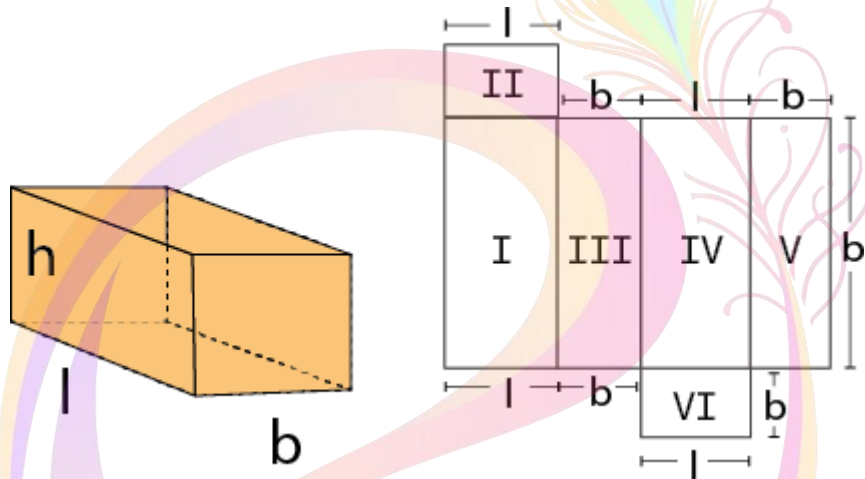
Solids with a Pair or More of Identical Faces



Surface Area of Solid Shapes

The surface area of the object is the total area occupied by the surface of the object. or surface area is simply the sum of the areas of the flat surfaces (called faces).

Surface Area of a Cuboid

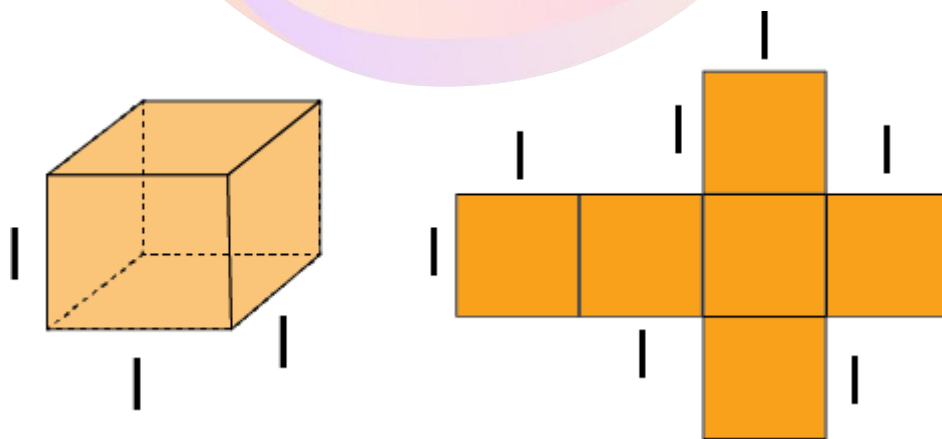


Total Surface area of cuboid = $2(lb+bh+lh)$

Lateral Surface area of cuboid = $2h(l+b)$

Where, l is the length, b is the breadth and h is the height.

Surface Area of a Cube

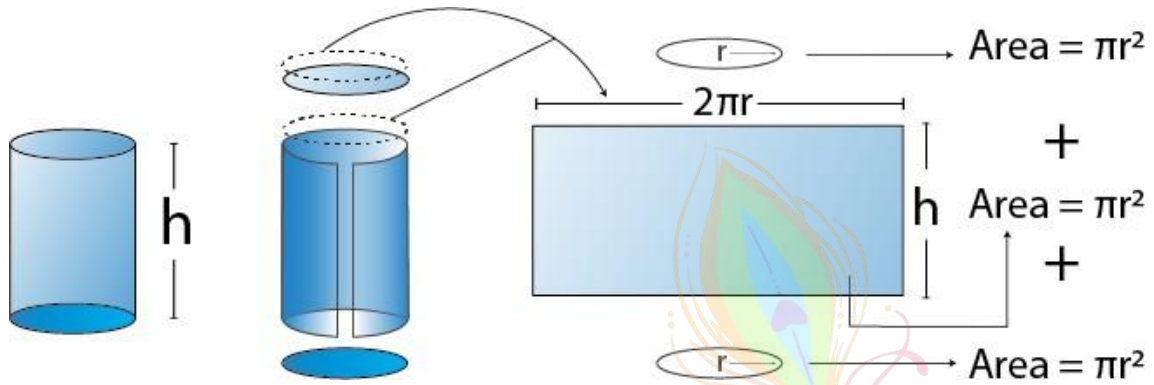


Total Surface area of a cube $=6l^2$

Lateral Surface area of a cube $=4l^2$

Where l is the length of each side of the cube.

Surface Area of a Cylinder



Curved surface area of cylinder (C.S.A) $=2\pi rh$

Total Surface area of cylinder (T.S.A) $=2\pi r(r+h)$

Where, r is the radius of the cylinder and h is the height of the cylinder.

Relation between Volume and Capacity

Volume is the total space occupied by an object. Volume is measured in cubic units,

Capacity refers to the maximum measure of an object's ability to hold a substance, like a solid, a liquid or a gas. Capacity can be measured in almost every other unit, including liters, gallons, pounds, etc.

E.g.: A bucket contains 9 litres of water, then its capacity is 9 litres.