NOTES

– NOTES WITH MIND MAPS –

MATHEMATICS

(CUBES AND CUBE ROOTS)

8^{TH}

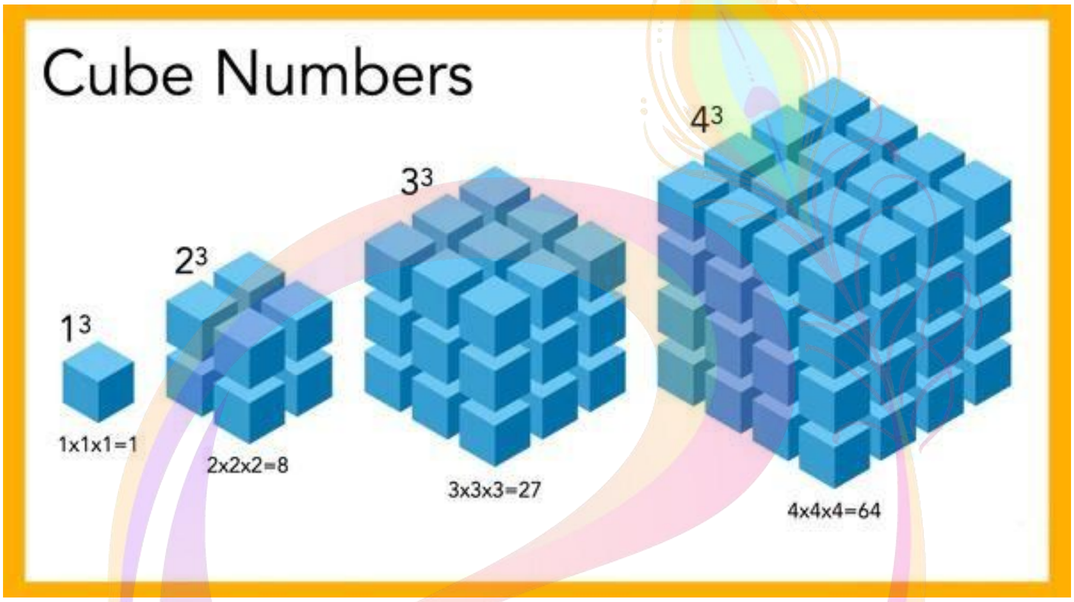
Cubes and Cube Roots

Cube Numbers

If a natural number m can be expressed as n3, where n is also a natural number, then m is called the cube number of n.

Numbers like 1, 8, 27 are cube number of the numbers 1, 2, and 3 respectively.

All perfect cube numbers are obtained by multiplying a number by itself three times.



Cubes Relation with Cube Numbers

In geometry, a cube is a solid figure where all edges are equal and are perpendicular to each other.

For example, take a cube of unit side. If we arrange these cubes to form a bigger cube of side 3 units, we find that there are a total of 27 such unit cubes that make up a cube of 3 units. Similarly, a cube of 4 units will have 64 such unit cubes.

Units Digits in Cube Numbers

Depending on whether a number is odd or even, its cube number is also odd or even respectively.

This is determined by the nature of the cube numbers' unit digit.

- If a number is odd, its cube numbers' unit digit is also odd.
- If a number is even, its cube numbers' unit digit is also even.

The table below shows the units digit of a number and the units digit of the cube of that number:

Units digit of number	Units digit of its cube
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

Inside Cube Numbers

Adding Consecutive Odd Numbers

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

We can see from the above pattern, if we need to find the n^3, n consecutive odd numbers will be needed, such that their sum is equal to n^3.

This pattern holds true for all natural numbers.

Also, if we need to find n3 then we should add n consecutive natural numbers starting from$\left(\frac{\frac{(n-1)(n)}{2}}{1}\right)^{th}$odd natural number.

Prime Factorization Method to Find a Cube

In the prime factorization of any number, if each prime factor appears three times, then the number is a perfect cube.

Consider, the number 216. By prime factorization,

$216=2\times2\times2\times3\times3\times3=2^3\times3^3=6^3$

Hence, 216 is a perfect cube.

Consider, the number 500. By prime factorization,

$500=2\times2\times5\times5\times5=2^2\times5^3$

In the above prime factorization 2 appears twice.

Hence, 500 is not a perfect cube.

Cube Roots

Finding the cube root is the inverse operation of finding the cube.

We know that $3^3=27$. We can also write the same equation as $\sqrt[3]{27}=3$. The symbol '3√'denotes 'cube root'.

Cube Root of 2

Let us consider another example of number 2. Since 2 is not a perfect cube number. It is not easy to find the cube root of 2. With the help of the long division method, it is possible to find the cube roots for non-perfect cube numbers. The approximate value of the $\sqrt[3]{2}$ is 1.260.

We can estimate the $\sqrt[3]{2}$ by using the trick here.

Since, 2 = 1 x 1 x 2

Cube root of 2 is approximately equal to (1 + 1+2)/3 = 4/3 = 1.333..

Cube root of 4

Again 4 is a number, which is not a perfect cube. If we factorise it, we get:

4 = 2 x 2 x 1

Hence, we can see, we cannot find the cube root by simple factorisation here.

Again, if we use the shortcut method, we get:

$\sqrt[3]{4}$ is equal to (2+2+1)/3 = 1.67

The actual value of $\sqrt[3]{4}$ is 1.587, which is approximately equal to 1.67.

Cubes and Cube Roots List of 1 to 15

Number	Cube(a^3)	Cube root $\sqrt[3]{a}$
1	1	1.000
2	8	1.260
3	27	1.442
4	64	1.587
5	125	1.710
6	216	1.817
7	343	1.913
8	512	2.000
9	729	2.080
10	1000	2.154
11	1331	2.224
12	1728	2.289
13	2197	2.351
14	2744	2.410
15	3375	2.466

Cube Root of 64

Since 64 is a perfect cube of 4, therefore, it is easy to find its cube-root by the prime factorisation method.

64 = 2 x 2 x 2 x 2 x 2 x 2

$\sqrt[3]{64} = \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 2)}$

= 2 x 2

= 4

Cube Root of 216

Since, 216 is perfect cube of 6, hence we can find the cube root of 216 by factorisation.

216 = 2 x 2 x 2 x 3 x 3 x 3

$\sqrt[3]{216} = \sqrt[3]{(2 \times 2 \times 2 \times 3 \times 3 \times 3)}$

$\sqrt[3]{216} = 2 \times 3$

$\sqrt[3]{216} = 6$

Cube Root of 343

Let us find the cube root of 343 with the help of the the prime factorisation method.

Dividing 343 by smallest prime factor, till we get the remainder as 1. Follow the below steps;

7	343
7	49
7	7
	1

Therefore, 343 = 7 × 7 × 7

And, $\sqrt[3]{343} = 7$

Cube Root of 512

To find the cube root of 512 we have tofactorise it first.

The prime factorisation of 512 can be written as:

512 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

Taking the cube roots both the sides, we get;

$\sqrt[3]{512} = \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)}$

$\sqrt[3]{512} = 2 \times 2 \times 2$

$\sqrt[3]{512} = 8$

Cube Root of 729

Now, let's find the cubic root of 729.

3	729
3	243
3	81
3	27
3	9
3	3
	1

Cube root of 729

729 = 3 × 3 × 3 × 3 × 3 × 3 = 9 × 9 × 9

Therefore, the cube root of 729 i.e. $\sqrt[3]{729} = 9$

Cube root using prime factorization

We can find the cube root of a number by prime factorization method by the following steps:

resolve the number into its prime factors. Consider the number 5832. 5832=(2×2×2)×(3×3×3)×(3×3×3).

make groups of three same prime factors.

take one prime factor from each group and multiply them. Their product is the required cube root.

Therefore, cube root of 5832=$\sqrt[3]{5832}$=2×3×3=18

Cube Root of a Cube Number using estimation

If a cube number is given we can find out its cube root using the following steps:

Take any cube number say 117649 and start making groups of three starting from the rightmost digit of the number. So 117649 has two groups, and first group(649) and the second group(117).

The unit's digit of the first group (649) will decide the unit digit of the cube root. Since the number 649 ends with 9, the cube roots unit's digit is 9.

Find the cube of numbers between which the second group lies. The other group is 117. We know that 4^3=64 and5^3=125. 64 < 117 < 125. Take the smaller number between 4 and 5 as the ten's digit of the cube root. So, 49 is the cube root of 117649.

Differences of Squares of Triangular Numbers and Converse

Triangular numbers: It is a sequence of the numbers 1, 3, 6, 10, 15 etc. It is obtained by continued summation of the natural numbers. The dot pattern of a triangular number can be arranged as triangles.

Sum of two consecutive triangular numbers gives us a square number. For example, $1+3=4=2^2$ and $3+6=9=3^2$.

The difference between the squares of two consecutive triangular numbers is a cube number. For example, $3^2 – 1^2=9 – 1=8=2^3$ and $6^2 – 3^2=3^6 – 9=2^5=5^3$.

Also, if the difference between the squares of two numbers is a cube number, then these numbers are consecutive triangular numbers.