NOTES

- NOTES WITH MIND MAPS -

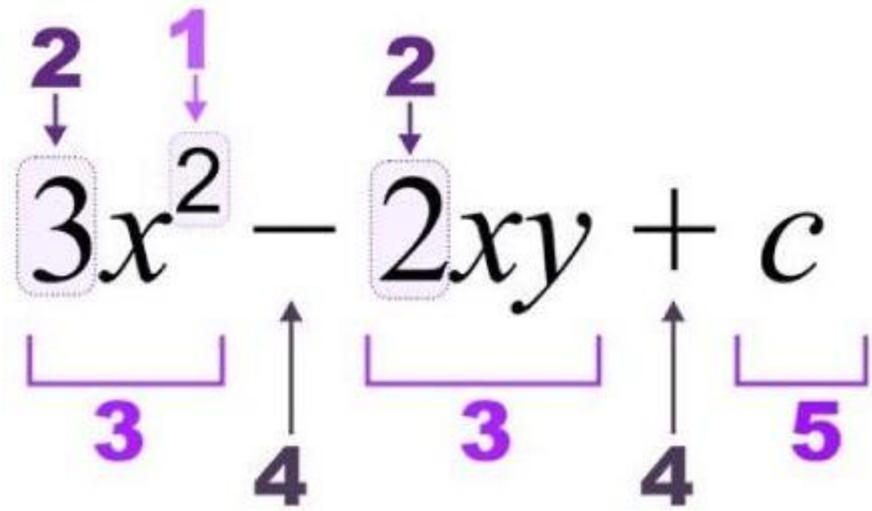
MATHEMATICS

(ALGEBRA)

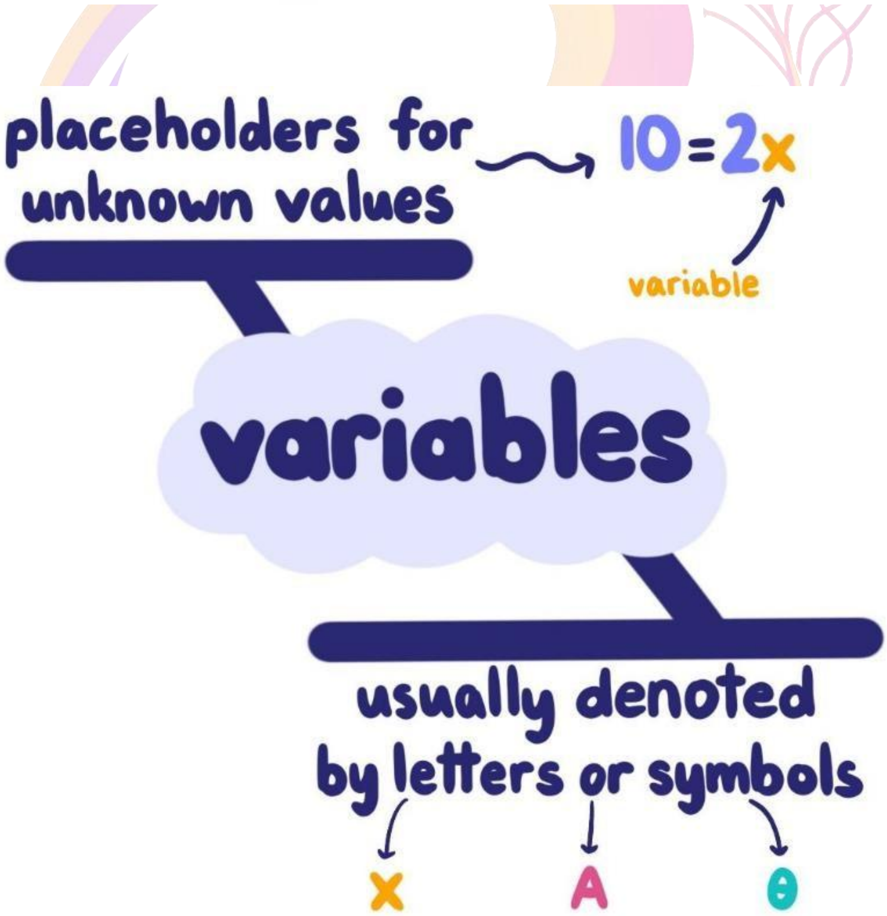
6 TH

ALGEBRA

Algebra is a branch of mathematics that can substitute letters for numbers to find the unknown. It can also be defined as putting real-life variables into equations and then solving them. The word Algebra is derived from Arabic "al-jabr", which means the reunion of broken parts. Below are some algebra problems for students to practice.



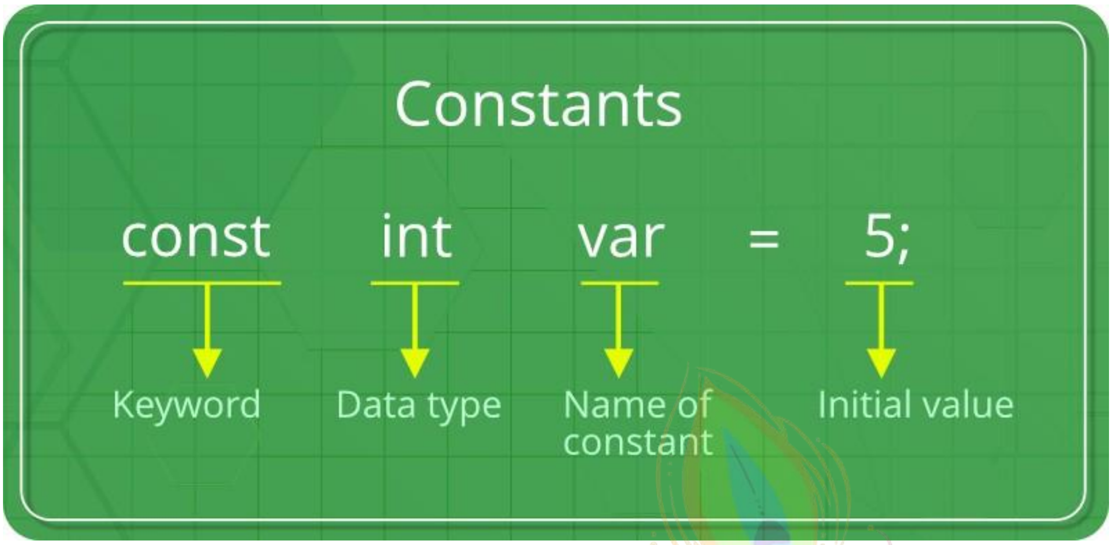
Variable



A variable is an unknown quantity that is prone to change with the context of a situation.

Example: In the expression 2x + 5, x is the variable.

Constant



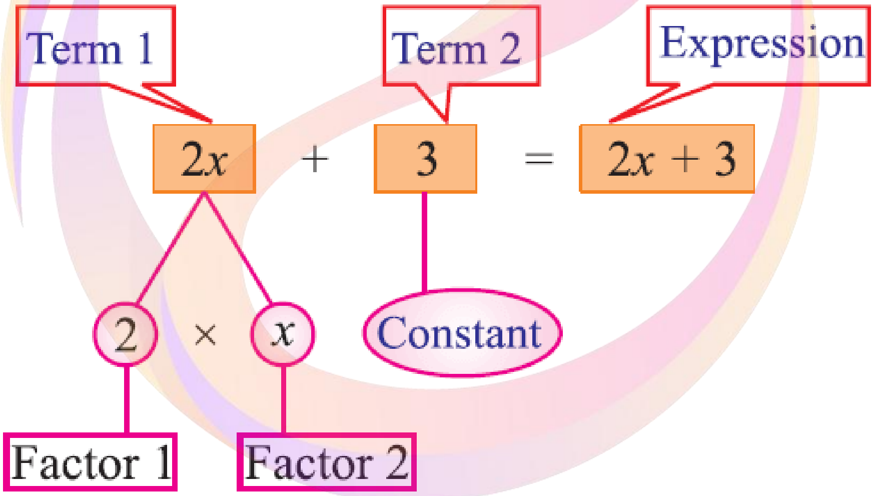
Constant is a quantity which has a fixed value. In the given example 2x + 5, 5 is the constant.

Terms of an Expression

Parts of an expression which are formed separately first and then added or subtracted, are known as terms.

In the above-given example, terms 2x and 5 are added to form the expression (2x + 5).

Factors of a term

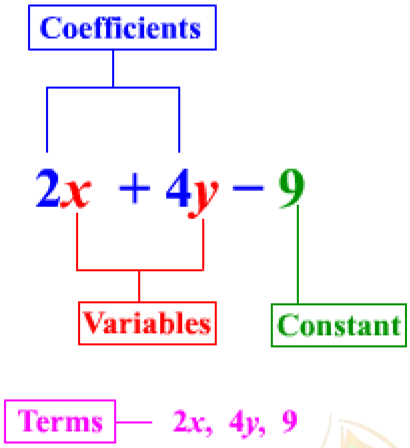


Parts of an expression which are formed separately first and then added or subtracted, are known as terms.

Factors of a term are quantities which cannot be further factorized.

In the above-given example, factors of the term 2x are 2 and x.

Coefficient of a term



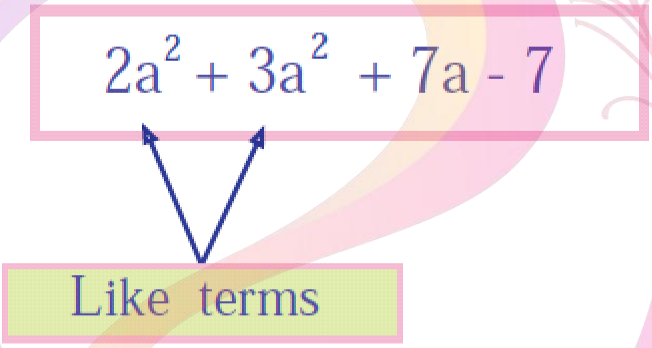
The numerical factor of a term is called the coefficient of the term.

In the above-given example, 2 is the coefficient of the term 2x.

To know more about Term, factor and Coefficient,

Like and Unlike Terms

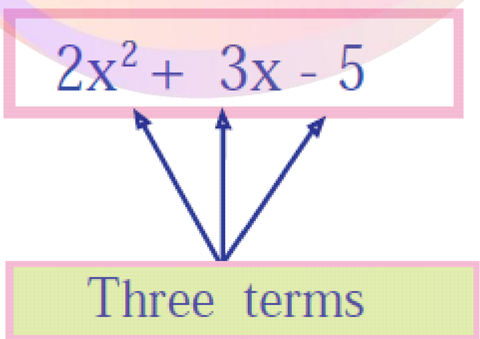
Like terms



Terms having the same variables are called like terms.>

Example: 8xy and 3xy are like terms.

Unlike terms



Terms having different variables are called, unlike terms.

Example: 7xy and -3x are unlike terms.

To know more about "Algebra Basics",

Monomial, Binomial, Trinomial and Polynomial Terms

Name	Monomial	Binomial	Trinomial	Polynomial
No. of terms	1	2	3	>3
Example	7xy	(4x−3)	(3x+5y−6)	(6x+5yx−3y+4)

Formation of Algebraic Expressions

Algebraic Expressions

An **algebraic expression** is a set of terms that are combined using addition (+), subtraction (-), multiplication (x) and division (÷)

An expression that contains two terms is called a **binomial**.

E.g. $2x + 3y$ or $2 - 5y^2$ etc.

An expression that contains three terms is called a **trinomial**.

E.g. $2x + 3y - 5$ or $2 - 5y^2 + 6xy$ etc.

Combinations of variables, constants and operators constitute an algebraic expression.

Example: 2x + 3, 3y + 4xy, etc.

To know more about Algebraic Expressions,

Addition and Subtraction of Algebraic Expressions

Addition and Subtraction of like terms

Sum of two or more like terms is a like term.

Its numerical coefficient will be equal to the sum of the numerical coefficients of all the like terms.

Example: 8y + 7y =?

8y
+7y
____
(8+7)y = 15y
____

Difference between two like terms is a like term.

Its numerical coefficient will be equal to the difference between the numerical coefficients of the two like terms.

Example: $11_z - 8_z = ?$

11z

-8z

____

(1-8)z = 3z

____

Addition and Subtraction of unlike terms

For adding or subtracting two or more algebraic expressions, like terms of both the expressions are grouped together and unlike terms are retained as it is.

Addition of $-5x^2 + 12xy$ and $7x^2 + xy + 7x$ is shown below:

$-5x^2+12xy$
$7x^2+xy+7x$

____

$2x^2+13xy+7x$

____

Subtraction of $-5x^2 + 12xy$ and $7x^2 + xy + 7x$ is shown below:

$-5x^2+12xy$
$-7x^2+xy+7x$

____

$12x^2+11xy-7x$

____

Algebra as Patterns

To understand the relationship between patterns and algebra, we need to try making some patterns. We can use pencils to construct a simple pattern and understand how to create a general expression to describe the entire pattern. It would be best if you had a lot of pencils for this. It will help if they are of similar height.

Find a solid surface and arrange two pencils parallel to each other with some space in between them. Add a second layer on top of it and another on top of that, as shown in the image given below.

There are a total of six pencils in this arrangement. The above arrangement contains three layers, and each layer has a fixed count of two pencils. The number of pencils in each layer never varies, but the number of layers you wish to build is entirely up to you.

Current Number of Layers = 4

Number of Pencils per Layer = 2

Total Number of Pencils = 2 × 4 = 8

What if you increase the number of layers to 10? What if you keep building up to a layer of 100? Can you sit and stack those many layers? Here, the answer is obviously NO. Instead, let's try to calculate.

Number of Layers = 100

Number of Pencils per Layer = 2

Total Number of Pencils = 2 × 100 = 200

There is an obvious pattern here. A single level has 2 pencils, which is always constant, regardless of the number of levels built. So to get the total number of pencils, we have to multiply 2 (the number of pencils per level) with the number of levels built. For example, to construct 30 levels, you will need 2 multiplied 30 times which is 60 pencils.

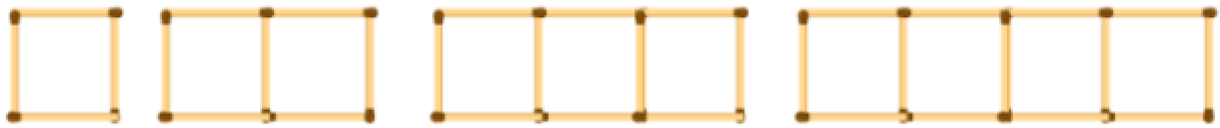
According to the previous calculation, to make a building of 'x' number of levels, we will require 2 multiplied 'x' times, and thus the number of pencils equal to 2x. We just created algebraic expressions based on patterns. In this way, we can make several algebra patterns.

Algebra as generalized Arithmetic Patterns

There are different types of algebraic patterns such as repeating patterns, growth patterns, number patterns, etc. All these patterns can be defined using different techniques. Let's go through the algebra patterns using matchsticks given below.

Algebra Matchstick Patterns

It is possible to make patterns with very basic things that we are using in our everyday life. Look at the following matchstick pattern of squares in the below figure. The squares are not separate. Two neighbouring squares have a common matchstick. Let's observe the patterns and try to find the rule that gives the number of matchsticks.



In the above matchstick pattern, the number of matchsticks is 4, 7, 10 and 13, which is one more than the thrice of the number of squares in the pattern.

Therefore, this pattern can be defined using the algebraic expression 3x + 1, where x is the number of squares.

Now, let's make the triangle pattern using matchsticks as shown in the below figure. Here, the triangles are connected with each other.

In this matchstick pattern, the number of matchsticks is 3, 5, 7 and 9, which is one more than twice the number of triangles in the pattern. Therefore, the pattern is 2x + 1, where x is the number of triangles.

Number patterns

If a natural number is denoted by n, then its successor is (n + 1).

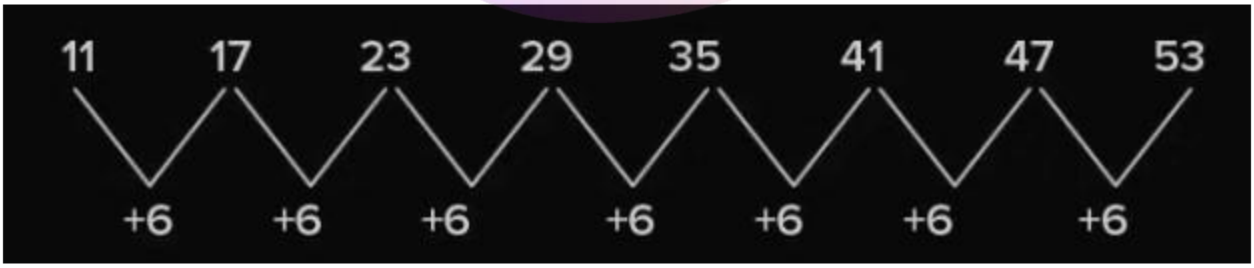
Example: Successor of n = 10 is n + 1 = 11.

If a natural number is denoted by n, then 2n is an even number and (2n + 1) is an odd number.

Example: If n = 10, then 2n = 20 is an even number and 2n+1=21 is an odd number.

Number Pattern Example

Consider an example given here. The given sequence of numbers is 11, 17, 23, 29, 35, 41, 47, and 53. The following figure helps to understand the relationship between the numbers.



In the given pattern, the sequence is increased by 6. It means the addition of the number 6 to the previous number gives the succeeding number. Also, the difference between the two consecutive number is 6.

Number Patterns Using Dots

Let's start representing each whole number with a set of dots and arranging these dots in some elementary shape to find number patterns. For arranging these dots, we take strictly four shapes into account. Numbers can be arranged into:

- A line
- A rectangle
- A square
- A triangle

Line

Every number can be arranged in a line. Examples:

The number 2 can be represented by 

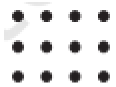
The number 3 can be represented by 

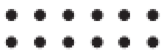
All other numbers can be represented in a similar pattern.

Rectangle

Some numbers can be arranged as a rectangle. Examples:

The number 6 can be arranged as a rectangle with 2 rows and 3 columns as 

Similarly, 12 can be arranged as a rectangle with 3 rows and 4 columns as 

Or as a rectangle with 2 rows and 6 columns as 

Similar it can be formed by 8, 10, 14, 15, etc.

Square

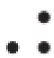
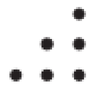
Some numbers can be arranged as squares. Examples:

The number 4 can be represented as  and 9 as 

Similar it can be formed by 16, 25, 36, 49 and so on.

Triangle

Some numbers can be arranged as triangles. Examples:

The number 3 can be represented as  and 6 as 

Similar it can be formed by 10, 15, 21, 28, etc. It is to be noted that the triangle should have its 2 sides equal. Hence, the number of dots in the rows starting from the bottom row should be like 4,3,2,1. The top row should always have one dot.

Number Pattern Types

There are different types of number patterns in Mathematics. They are:

- Arithmetic Sequence
- Geometric Sequence
- Square Numbers
- Cube Numbers
- Triangular Numbers
- Fibonacci Numbers
- Number Patterns Observation

Observation of number patterns can guide to simple processes and make the calculations easier.

Consider the following examples which help the addition and subtraction with numbers like 9, 99, 999, etc. simpler.

145 + 9 = 145 + 10 – 1 = 155 – 1 = 154

145 – 9 = 145 – 10 + 1 = 135 + 1 = 136

145 + 99 = 145 + 100 – 1 = 245 – 1 = 244

145 – 99 = 145 – 100 + 1 = 45 + 1 = 46

Consider another pattern which simplifies multiplication with 9, 99, 999, and so on:

62 x 9 = 62 x (10 – 1) = 558

62 x 99 = 62 x (100 – 1) = 6138

62 x 999 = 62 x (1000-1) = 61938

Consider the following pattern which simplifies multiplication with numbers like 5, 25, 15, etc.:

48 x 5 = 48 x 10/2 = 480/2 = 240

48 x 25 = 48 x 100/4 = 4800/4 = 1200

48 x 125 = 48 x 1000/8 = 48000/8 = 6000

A number of patterns of similar fashion can be observed in the whole numbers which simplifies calculations to quite an extent.