NOTES

- NOTES WITH MIND MAPS -

# MATHEMATICS

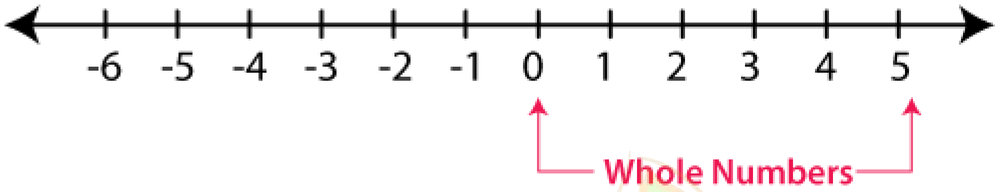
## (INTEGERS)

6TH

# INTEGERS

## Whole Numbers

Whole numbers include zero and all natural numbers i.e. 0, 1, 2, 3, 4, and so on.




The whole numbers are the numbers without fractions and it is a collection of positive integers and zero. It is represented by the symbol "W" and the set of numbers are {0, 1, 2, 3, 4, 5, 6, 7, 8, 9,............ }. Zero as a whole represents nothing or a null value.

- Whole Numbers: W = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10......}
- Natural Numbers: N = {1, 2, 3, 4, 5, 6, 7, 8, 9,...}
- Integers: Z = {....-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,...}
- Counting Numbers: {1, 2, 3, 4, 5, 6, 7,....}

These numbers are positive integers including zero and do not include fractional or decimal parts (3/4, 2.2 and 5.3 are not whole numbers). Addition, Subtraction, Multiplication and Division operations are possible on whole numbers.

## Symbol

The symbol to represent whole numbers is the alphabet 'W' in capital letters.

W = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,...}

Thus, the whole numbers list includes 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ....

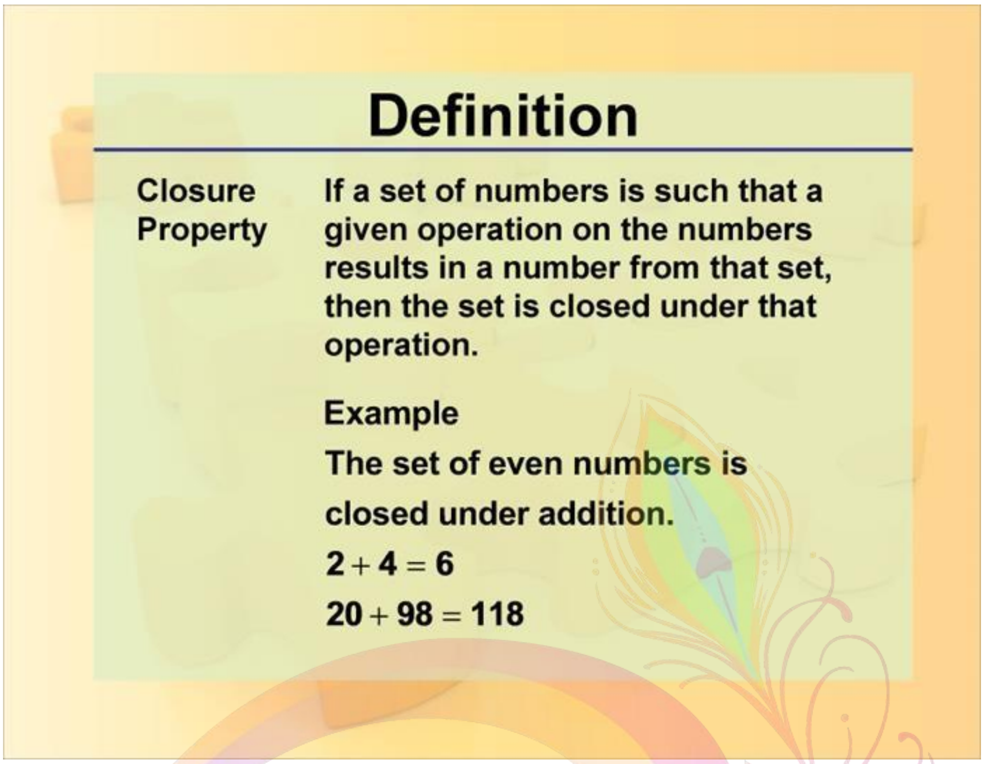
## Facts:

- All the natural numbers are whole numbers
- All counting numbers are whole numbers
- All positive integers including zero are whole numbers
- All whole numbers are real numbers

## Whole Numbers Properties

The properties of whole numbers are based on arithmetic operations such as addition, subtraction, division and multiplication. Two whole numbers if added or multiplied will give a whole number itself. Subtraction of two whole numbers may not result in whole numbers, i.e. it can be an integer too. Also, division of two whole numbers results in getting a fraction in some cases. Now, let us see some more properties of whole numbers and their proofs with the help of examples here.

**Closure Property**




They can be closed under addition and multiplication, i.e., if x and y are two whole numbers then x. y or x + y is also a whole number.

**Example:**

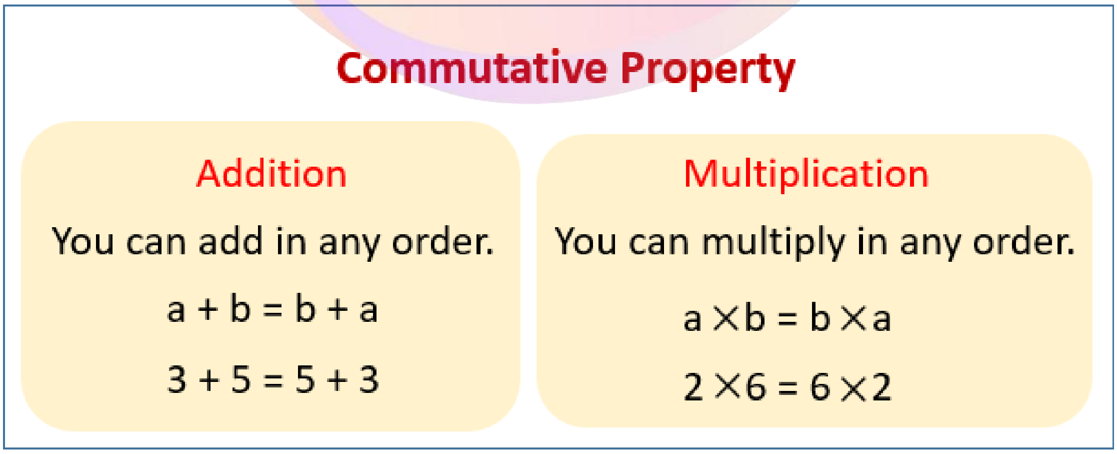
5 and 8 are whole numbers.

5 + 8 = 13; a whole number

5 × 8 = 40; a whole number

Therefore, the whole numbers are closed under addition and multiplication.

## Commutative Property of Addition and Multiplication




The sum and product of two whole numbers will be the same whatever the order they are added or multiplied in, i.e., if x and y are two whole numbers, then x + y = y + x and x.y = y.x

**Example:**

Consider two whole numbers 3 and 7.

3 + 7 = 10

7 + 3 = 10

Thus, 3 + 7 = 7 + 3.

Also,

3 × 7 = 21

7 × 3 = 21

Thus, 3 × 7 = 7 × 3

Therefore, the whole numbers are commutative under addition and multiplication.

## Additive identity

| Additive Identity |
|---|
| Used for addition operation |
| Given by: p + 0 = p = 0 + p |
| 0 is the identity element |
| For example, 77 + 0 = 77, -77 + 0 = -77 |

When a whole number is added to 0, its value remains unchanged, i.e., if x is a whole number then x + 0 = 0 + x = x

**Example:**

Consider two whole numbers 0 and 11.

0 + 11 = 0

11 + 0 = 11

Here, 0 + 11 = 11 + 0 = 11

Therefore, 0 is called the additive identity of whole numbers.

## Multiplicative identity

| Multiplicative Identity |
|---|
| Used for multiplication operation |
| Given by: $p \times 1 = p = 1 \times p$ |
| 1 is the identity element |
| For example, $77 \times 1 = 77$,<br>$-77 \times 1 = -77$ |

When a whole number is multiplied by 1, its value remains unchanged, i.e., if x is a whole number then x.1 = x = 1.x

**Example:**

Consider two whole numbers 1 and 15.

$1 \times 15 = 15$

$15 \times 1 = 15$

Here, $1 \times 15 = 15 = 15 \times 1$

Therefore, 1 is the multiplicative identity of whole numbers.

**Associative Property**

**Associative Property Formula**

$$a + (b + c) = (a + b) + c$$

$$a \times (b \times c) = (a \times b) \times c$$

When whole numbers are being added or multiplied as a set, they can be grouped in any order, and the result will be the same, i.e. if x, y and z are whole numbers then x + (y + z) = (x + y) + z and x. (y.z) = (x.y). z

**Example:**

Consider three whole numbers 2, 3, and 4.

2 + (3 + 4) = 2 + 7 = 9

(2 + 3) + 4 = 5 + 4 = 9

Thus, 2 + (3 + 4) = (2 + 3) + 4

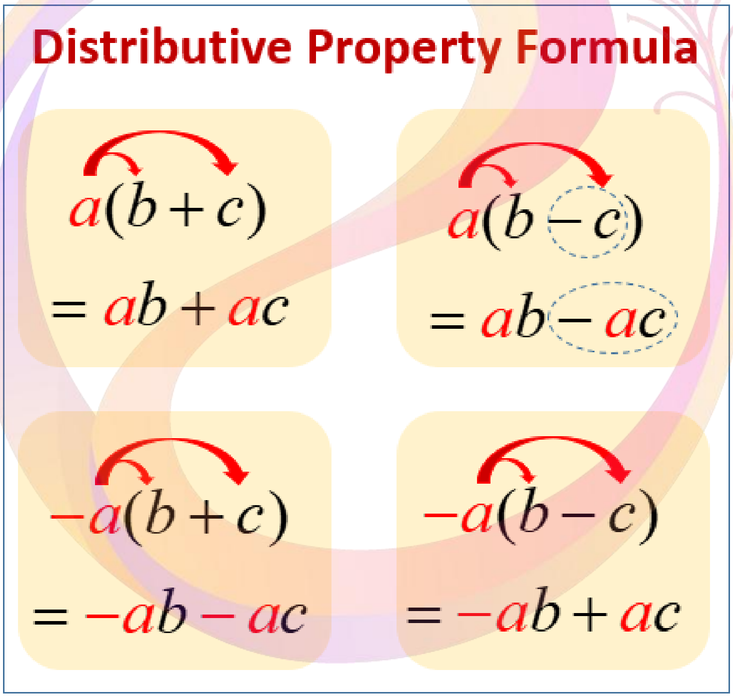
2 × (3 × 4) = 2 × 12 = 24

(2 × 3) × 4 = 6 × 4 = 24

Here, 2 × (3 × 4) = (2 × 3) × 4

Therefore, the whole numbers are associative under addition and multiplication.

**Distributive Property**




If x, y and z are three whole numbers, the distributive property of multiplication over addition is x. (y + z) = (x.y) + (x.z), similarly, the distributive property of multiplication over subtraction is x. (y – z) = (x.y) – (x.z)

**Example:**

Let us consider three whole numbers 9, 11 and 6.

9 × (11 + 6) = 9 × 17 = 153

(9 × 11) + (9 × 6) = 99 + 54 = 153

Here, 9 × (11 + 6) = (9 × 11) + (9 × 6)

Also,

9 × (11 – 6) = 9 × 5 = 45

(9 × 11) – (9 × 6) = 99 – 54 = 45

So, 9 × (11 – 6) = (9 × 11) – (9 × 6)

Hence, verified the distributive property of whole numbers.

## Multiplication by zero


When a whole number is multiplied to 0, the result is always 0, i.e., x.0 = 0.x = 0

**Example:**

0 × 12 = 0

12 × 0 = 0

Here, 0 × 12 = 12 × 0 = 0

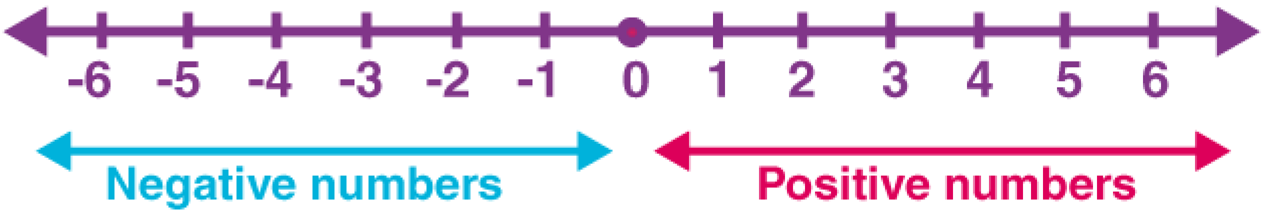
Thus, any whole number multiplied by 0, the result is always 0.

## Division by zero

Division of a whole number by o is not defined, i.e., if x is a whole number, then x/0 is not defined.

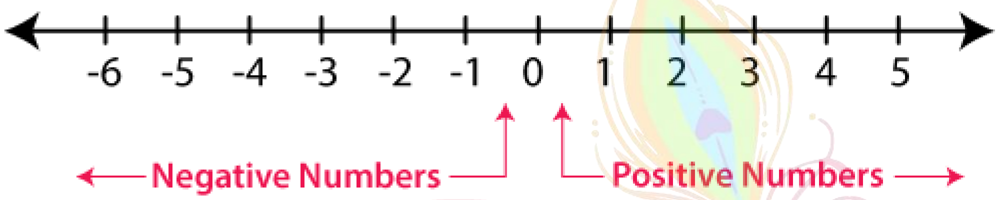
## Numbers on a Number Line

Arithmetic operations of numbers can be better explained on a number line. To begin with, one must know to locate numbers on a number line. Zero is the middle point of a number line. All (natural numbers) positive numbers occupy the right side of the zero whereas negative numbers occupy the left side of zero on the number line. As we move on to the left side value of a number decreases. For example, 1 is greater than -2. In a number line, integers, fractions, and decimals can also be represented easily. Check out the links given below to learn more.




**Negative Numbers**

The numbers with a negative sign and which lies to the left of zero on the number line are called negative numbers.




**Applications of Negative Numbers in Real Life**

Irrespective of their value, they have a broad connection to daily life. These numbers are widely used in different fields. Some of the real-life examples are given below.

**Finance and Banking**

Banking and financing are all about money, credit and debit. Hence, we need some numbers which can differentiate a credit amount from the debit amount. Another instance is profit and loss. These all are mathematically denoted by using positive and negative integers. If someone debited to someone it is represented by a minus sign. The stock market is another field which widely uses negative integers to show its share price and ups and downs.

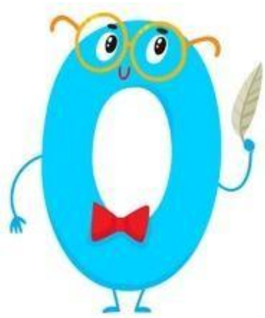
**Science**

Use of negative numbers is commonly observed in weather broadcasting. Thermometers are vertical number lines which measure the temperature of a body as well as the temperature of an area. Meteorologist uses negative numbers to show the cold condition of a region like -15˚C. Even when the body temperature goes down a negative integer is used to represent the condition. Temperature below zero denoted with the negative sign while a temperature above zero denoted with the positive sign. Other instruments and conditions that depend on integers are batteries, blood pressure, overweight and underweight, drug testing and so on.

**Other Applications**

In sports, the goal differences in games like hockey, football are denoted by integers. Other examples are the speed of a car, the rating of songs or movies, the numbering of a story of a building, etc.

### Introduction to Zero



### The number Zero

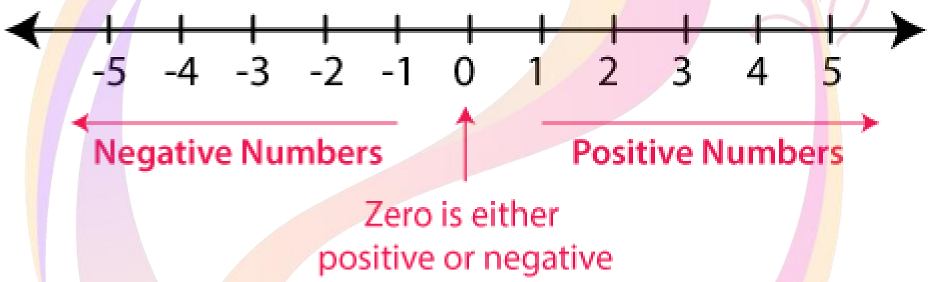
The number zero means an absence of value.

### The Number Line

### Integers

Collection of all positive and negative numbers including zero are called integers. ⇒ Numbers ..., – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, ... are integers.

### Representing Integers on the Number Line

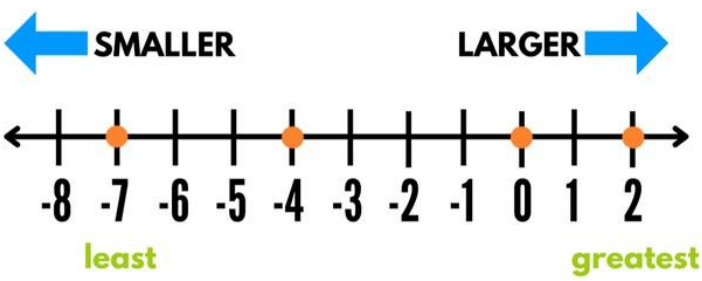



- Draw a line and mark a point as 0 on it
- Points marked to the left (-1, -2, -3, -4, -5, -6) are called negative integers.
- Points marked to the right (1, 2, 3, 4, 5, 6) or (+1, +2, +3, +4, +5, +6) are called positive integers.

## Absolute value of an integer

- Absolute value of an integer is the numerical value of the integer without considering its sign.
- Example: Absolute value of -7 is 7 and of +7 is 7.

## Ordering Integers



- On a number line, the number increases as we move towards right and decreases as we move towards left.
- Hence, the order of integers is written as..., –5, –4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5...
- Therefore, – 3 < – 2, – 2 < – 1, – 1 < 0, 0 < 1, 1 < 2 and 2 < 3.

## Addition of Integers

**Positive integer + Negative integer**

Example: (+5) + (-2) Subtract: 5 – 2 = 3 Sign of bigger integer (5): + Answer: +3

Example: (-5) + (2) Subtract: 5-2 = 3 Sign of the bigger integer (-5): – Answer: -3

**Positive integer + Positive integer**

Example: (+5) + (+2) = +7

Add the 2 integers and add the positive sign.

**Negative integer + Negative integer**

Example: (-5) + (-2) = -7

Add the two integers and add the negative sign.

## Properties of Addition and Subtraction of Integers

**Operations on Integers**

Operations that can be performed on integers:

- Addition
- Subtraction
- Multiplication
- Division.

**Subtraction of Integers**

The subtraction of an integer from another integer is same as the addition of the integer and its additive inverse.

Example: 56 – (–73) = 56 + 73 = 129 and 14 – (8) = 14 – 8 = 6

## Properties of Addition and Subtraction of Integers

**Closure under Addition**

a + b and a – b are integers, where a and b are any integers.

**Commutativity Property**

a + b = b + a for all integers a and b.