

NOTES

- NOTES WITH MIND MAPS -
MATHEMATICS
(PLAYING WITH NUMBERS)



PLAYING WITH NUMBERS



Factor

Factors of 276: 1, 2, 3, 4, 6, 12, 23, 46, 69, 92, 138, 276

Negative Factors of 276: -1, -2, -3, -4, -6, -12, -23, -46, -69, -92, -138, -276

Prime factorization of 276: $2 \times 2 \times 3 \times 23$

The factor of a number is an exact divisor of that number

Example

1. Factor of 6

1 → Since 1 exactly divides 6

2 → Since it exactly divides 6

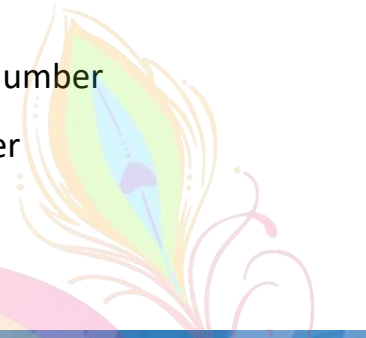
3 → Since it exactly divides 6

6 → Since it exactly divides 6

Properties of factors

- 1 is a factor of every number
- Every number is a factor of itself.
- Every factor of a number is an exact divisor of that number
- Every factor is less than or equal to the given number
- Number of factors of a given number are finite.

Multiple



Definition

Multiple	The product of any number or expression and an integer.
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Numerical Examples of Multiples:

Multiples of 2: 2, 4, 6, 8, ...

Multiples of 6: 6, 12, 18, 24, 30, ...

Algebraic Examples of Multiples:

Multiples of n : $n, 2n, 3n, 4n, \dots$

Multiples of $(x + 1)$: $(x + 1), 2(x + 1), 3(x + 1), \dots$

Multiple of a number is the numbers obtained by multiplying that numbers with various Natural numbers

Example

Number is 6

Multiple will be

$$6 \times 1 = 6$$

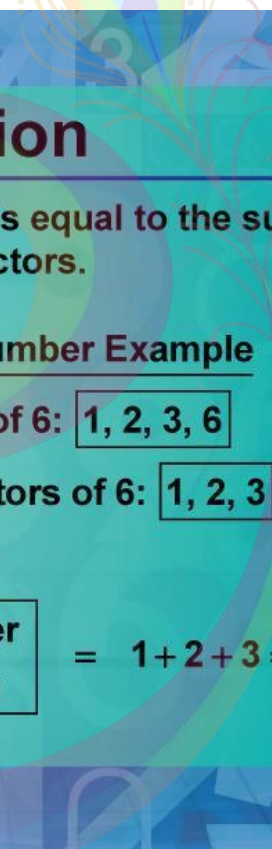
$$6 \times 2 = 12$$

$$6 \times 3 = 18$$

Properties of Multiple

- Every multiple of a number is greater than or equal to that number.
- Number of multiples of a given number is infinite.
- Every number is a multiple of itself.

Perfect Number



Definition

Perfect Number	A number that is equal to the sum of its proper factors.
Perfect Number Example	
Factors of 6: 1, 2, 3, 6	
Proper Factors of 6: 1, 2, 3	
Sum of Proper Factors of 6	= 1 + 2 + 3 = 6

A number for which sum of all its factors is equal to twice the number is called a perfect number

Example

1. 6

The factors of 6 are 1, 2, 3 and 6.

$$\text{Now, } 1 + 2 + 3 + 6 = 12 = 2 \times 6$$

2. 28

All the factors of 28 are 1, 2, 4, 7, 14 and 28.


$$\text{Now, } 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 \times 28$$

Prime Numbers

Definition

Prime Number	A positive integer with only two factors, 1 and itself. A number that is not composite.
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Examples of Prime Numbers:	Examples of Composite Numbers:
3 has factors 1, 3	4 has factors 1, 2, 4
5 has factors 1, 5	8 has factors 1, 2, 4, 8
13 has factors 1, 13	24 has factors 1, 2, 3, 4, 6, 8, 12, 24
23 has factors 1, 23	15 has factors 1, 3, 5, 15



The numbers other than 1 whose only factors are 1 and the number itself are called Prime numbers.

Example:

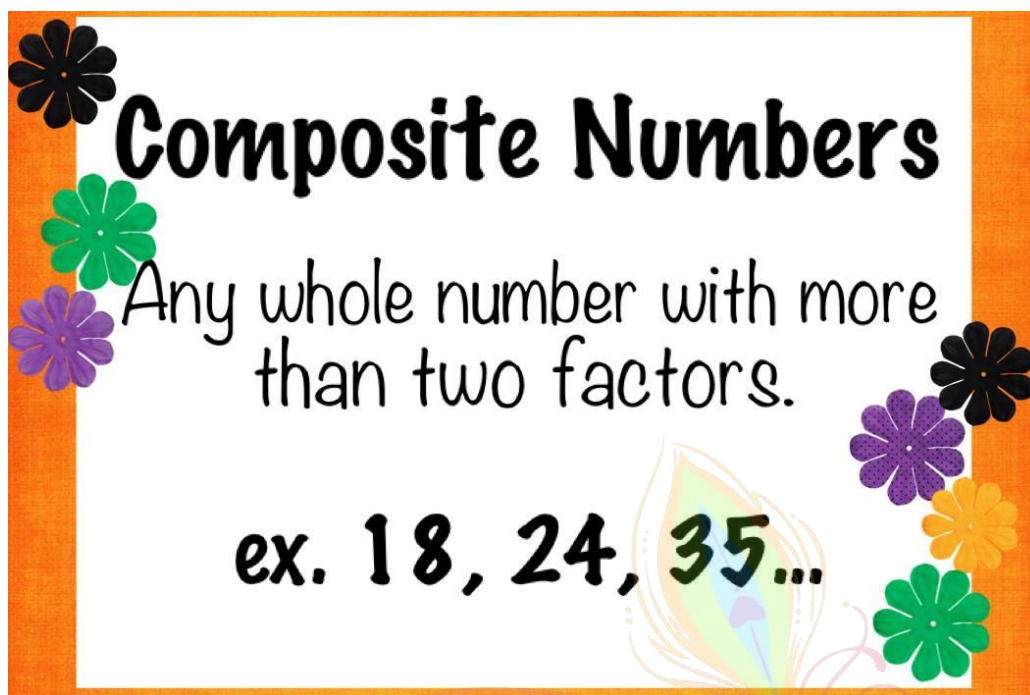
2, 3, 5, 7, 11, 13

We can find list of prime numbers till 100 using Sieve of Eratosthenes method

	②	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Step 1: Cross out 1 because it is not a prime number.
 - Step 2: Encircle 2, cross out all the multiples of 2, other than 2 itself, i.e., 4, 6, 8 and so on.
 - Step 3: You will find that the next uncrossed number is 3. Encircle 3 and cross out all the multiples of 3, other than 3 itself.
 - Step 4: The next uncrossed number is 5. Encircle 5 and cross out all the multiples of 5 other than 5 itself.
 - Step 5: Continue this process till all the numbers in the list are either encircled or crossed out.
- All the encircled numbers are prime numbers. All the crossed-out numbers, other than 1 are composite numbers

Composite Numbers



Numbers having more than two factors (1 and itself) are called Composite numbers

Example:

4, 6, 8, 9....

Zero (0) is considered as neither prime nor a composite number because it does not have any factors.

Composite Numbers Examples

The examples of composite numbers are 6, 14, 25, 30, 52, etc, such that:

Composite numbers	Factors
6	1, 2, 3, 6
14	1, 2, 7, 14
25	1, 5, 25
30	1, 2, 3, 5, 6, 10, 15, 30
52	1, 2, 4, 13, 26, 52

Properties of Composite Numbers

The properties of composite numbers are easy to remember.

- Composite numbers have more than two factors

- Composite numbers are evenly divisible by their factors
- Each composite number is a factor of itself
- The smallest composite number is 4
- Each composite number will include at least two prime numbers as its factors (E.g., $10 = 2 \times 5$, where 2 and 5 are prime numbers)
- Composite numbers are divisible by other composite numbers also

Even Numbers

Even Numbers									
Even Numbers Chart 1 - 100									
2	12	22	32	42	52	62	72	82	92
4	14	24	34	44	54	64	74	84	94
6	16	26	36	46	56	66	76	86	96
8	18	28	38	48	58	68	78	88	98
10	20	30	40	50	60	70	80	90	100

The numbers which are multiple of 2 are called even numbers

Example

2, 4, 6, 8, 10, 12, 14

Even numbers have 0, 2, 4, 6, 8 in it one's place

Odd Numbers

Odd Numbers Chart 1-100									
1	11	21	31	41	51	61	71	81	91
3	13	23	33	43	53	63	73	83	93
5	15	25	35	45	55	65	75	85	95
7	17	27	37	47	57	67	77	87	97
9	19	29	39	49	59	69	79	89	99

The numbers which are not multiple of 2 are called odd numbers

Example

1,3, 5,7,9,11.....

Important points about prime numbers based on definition of odd and even numbers

- 2 is the smallest prime number which is even.
- every prime number except 2 is odd.

Tests for Divisibility of Numbers

Number	Test of divisibility
2	A number is divisible by 2 if it has any of the digits 0, 2, 4, 6 or 8 in its one's place
3	A number is divisible by 3 if the sum of the digits is a multiple of 3, then the number is divisible by 3. Example 153 - Sum of digit = $1+5+3=9$ and $9/3=3$ So, 153 is divisible by 3
4	1) For one and two-digit number, just check the divisibility by actual division 2) For number with 3 or more digits is divisible by 4 if the number formed by its last two digits (i.e. ones and tens) is divisible by 4.
5	A number is divisible by 5 if a number which has either 0 or 5 in its one's place
6	A number is divisible by 6 if a number is divisible by 2 and 3 both
7	It must be checked by actual division
8	1) For one, two-digit number, three-digit and four-digit number, just check the divisibility by actual division 2) For a number with 4 or more digits is divisible by 8, if the number formed by the last three digits is divisible by 8
9	A number is divisible by 9, if the sum of the digits of a number is divisible by 9
10	A number is divisible by 10 if a number has 0 in the ones
11	A number is divisible by 11 if the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number is either 0 or divisible by 11,