

MATH



CHAPTER 8: FACTORS AND MULTIPLES

FACTORS AND MULTIPLES

LEARNING OBJECTIVES

This lesson will help you to:

- understand the concept of factors.
- understand the concept of multiples.
- use factor tree to find the factors of a number.
- to find prime numbers, factors and multiples of given number.
- to apply factors and multiples to real life situations.

❖ Amazing Fact

A number's composite factors are found by multiplying 2 or more prime factors.

For example: The composite factors of 18($2 \times 3 \times 3$) are 6(2×3) and 9(3×3)

Real Life Example

Money can use the concept of factors. One can exchange a 100 – Rupee by two 50 – Rupee notes (factors 2 and 50) or five 20 – Rupee note (factors 5 and 20).

QUICK CONCEPT REVIEW

Factors are numbers that multiplies to get another number.

For example: 4 and 7 are multiplied to get 28, then 4 and 7 are factors of 28.

Multiples are product obtained by multiplying one number by another.

For example: 8 and 11 are multiplied to get 88, then 88 is a multiple of 8 and 11.

The factors (or multiples) that are common between 2 or more numbers are called common factors (or multiples) of given numbers.

➤ PROPERTIES OF FACTORS AND MULTIPLIES

- 1 is a factor of every number.
- Every number is a factor of itself.
- Every factor of a number is an exact divisor of that number.
- Every factor of a number is less than or equal to the number.
- Factors of a given number are finite.
- Prime numbers have only 2 factors: 1 and the number itself.
- Every number is a multiple of itself.
- Every multiple of a number is greater than or equal to that number.
- The number of multiples of a given number is unlimited.

❖ Play Time

(1) Make two teams.

Ask the first team to pick up a number between 1 and 50. Then ask them to call out a factor of that number. Ask the second team to call out a factor or multiple of the called out number. Continue this process till all the factors and multiples are said. One who cannot give the factor or multiple will be out.

❖ Misconcept/Concept

Misconcept: Student might confuse between the concept of factors and multiples.

Concept: Explain factors come from dividing and multiples come from multiplying.

Misconcept: 1 is a prime number.

Concept: 1 is not a prime number even 1 is neither composite nor prime.

❖ Factors of a Number

All the numbers, which divide a certain number exactly, without leaving a remainder are called factors of that number.

For example:

$12 \div \textcircled{1} = 12$, $12 \div \textcircled{2} = 6$, $12 \div \textcircled{3} = 4$, $12 \div \textcircled{4} = 3$, $12 \div \textcircled{6} = 2$, $12 \div \textcircled{12} = 1$
 $\Rightarrow 1, 2, 3, 4, 6$ and 12 are factors of 12 .

Note: Factors of a number always include 1 and the number itself.

Example:

Find the factors of 15 .

Solution: The factor of 15 are

$15 \div \textcircled{1} = 15$, $15 \div \textcircled{3} = 5$, $15 \div \textcircled{5} = 3$, $15 \div \textcircled{15} = 1$
 $\Rightarrow 1, 3, 5$ and 15 are factors of 15 .

Example:

Which among the following is not a factor of 10 ?

- (a) 2
- (b) 5
- (c) 10
- (d) 3
- (e) None of these

Answer: (d)

Explanation:

Clearly $10 \div 1 = 10$, $10 \div 2 = 5$, $10 \div 5 = 2$ and $10 \div 10 = 1$

$\Rightarrow 1, 2, 5$ and 10 are factors of 10 .

❖ Properties of factors:

- (i) 1 is a factor of every number.
- (ii) Every non-zero number is a factor of itself.
- (iii) Every non-zero number is a factor of zero.
- (iv) Division by 0 is meaningless.
- (v) The factor of a non-zero number is either less than or equal to the number.

Example:

Which among the following statements is not true?

- (a) 2 is a factor of 2.
- (b) 26 is a factor of 0
- (c) 28 is not a factor of 4.
- (d) 4 is not a factor of 28
- (e) None of these

Answer: (d)

Explanation: Every number is a factor of itself so 2 is a factor of 2.

Every non-zero number is a factor of 0. So 26 is a factor of 0.

The factor of a non-zero number cannot be greater than the number.

So, 28 can't be a factor of 4.

$$28 = 1 \times 2 \times 2 \times 7$$

\Rightarrow 1, 2, 4, 7, 14 and 28 are factors of 28.

❖ Even and odd Numbers

Even numbers: A number is called an even number if 2 is a factor of the number. In other words, A number, which is a multiple of 2 is called an even number.

For example: 0, 2, 4, 6, 8, 10, 12, 14, 16 are even numbers.

Odd numbers: A number, which is not a multiple of 2 is called an odd number.

For example 1, 3, 5, 7, 9, 11, — are odd numbers.

Example:

Which one among the following is not an even number?

- (a) 0
- (b) 89990
- (c) 1049
- (d) 2032
- (e) None of these

Answer: (c)

Explanation: 1049 is not a multiple of 2 and so is not an even number.

❖ Prime Factors

Factors of a number written in primes are called prime factors of that number.

For example: $24 = 2 \times 2 \times 2 \times 3$

⇒ Prime factors of 24 are $2 \times 2 \times 2 \times 3$

❖ Multiples

You already know that multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20,.....

So all the numbers that comes in the table of 2 are its multiples. Multiples are never ending.

So it is not possible to find the last multiple of any number.

Example:

Write first five multiples of 8 and 16 and find common multiples of both.

Solution: First five multiples of 8 = 8, 16, 24, 32, 40

First five multiples of 16 = 16, 32, 48, 64, 80

Clearly common multiples from first five multiples of 8 and 16 are 16 and 32.

Example:

Which one among the following is not a multiple of 11?

- (a) 101
- (b) 121
- (c) 154
- (d) 176
- (e) None of these

Answer: (a)

Explanation:

$$121 = 11 \times 11, 154 = 11 \times 14$$

$$176 = 11 \times 16 \text{ and } 101 = 1 \times 101$$

Here 101 is a prime number and it is not a multiple of 11.

❖ **Properties of Multiples**

- (i) Every number is a multiple of 1.
- (ii) Every non-zero number is a multiple of itself.
- (iii) Multiples of any number are infinite.
- (iv) Every non-zero multiple of a non-zero number is either greater than or equal to the number.

❖ Highest Common Factor

The highest common factor among two or more given numbers is called the highest common factor or (H.C.F.).

Example:

Find the HCF of 12 and 16.

Solution:

$$\because 12 = 2 \times 2 \times 3 \text{ and } 16 = 2 \times 2 \times 2 \times 2$$

$$\because \text{HCF of } 12 \text{ and } 16 = 2 \times 2 = 4$$

❖ AlterNet Method

Factors of 12 = 1, 2, 4, 6, 12 $\{\because 12 = 1 \times 2 \times 2 \times 3\}$ and

Factors of 16 = 1, 2, 4, 8, 16 $\{\because 16 = 1 \times 2 \times 2 \times 2 \times 2\}$

$$\therefore \text{Highest common factors of } 12 \text{ and } 16 = 4$$

Lowest Common Multiple (L.C.M.)

Since multiples of a number are uncountable. So it is not possible to get the highest common multiple. Let us learn the steps used to find the lowest common multiple of two numbers.

Step 1: Find first few multiples of smaller number.

Step 2: Find first few multiples of larger number till we get a common multiple of both the numbers.

Step 3: The common multiple so obtained will be the lowest common multiple of both the numbers.

Example:

Find the lowest common multiple of 4 and 10.

Solution:

Step 1: First 6 multiples of 4 are 4, 8, 12, 16, 20, 24

Step 2: First 2 multiples of 10 are 10, 20

Step 3: Clearly 20 is the first common multiple of both the numbers.

So, 20 is the LCM of 4 and 10.

Alternate Method

First write both the numbers separated with a comma, and then find the prime factors of both the number as shown below.

2	4, 10
2	2, 5
5	1, 5
	1, 1

So LCM of 4 and 10 is $2 \times 2 \times 5 = 20$.