NOTES

– NOTES WITH MIND MAPS –

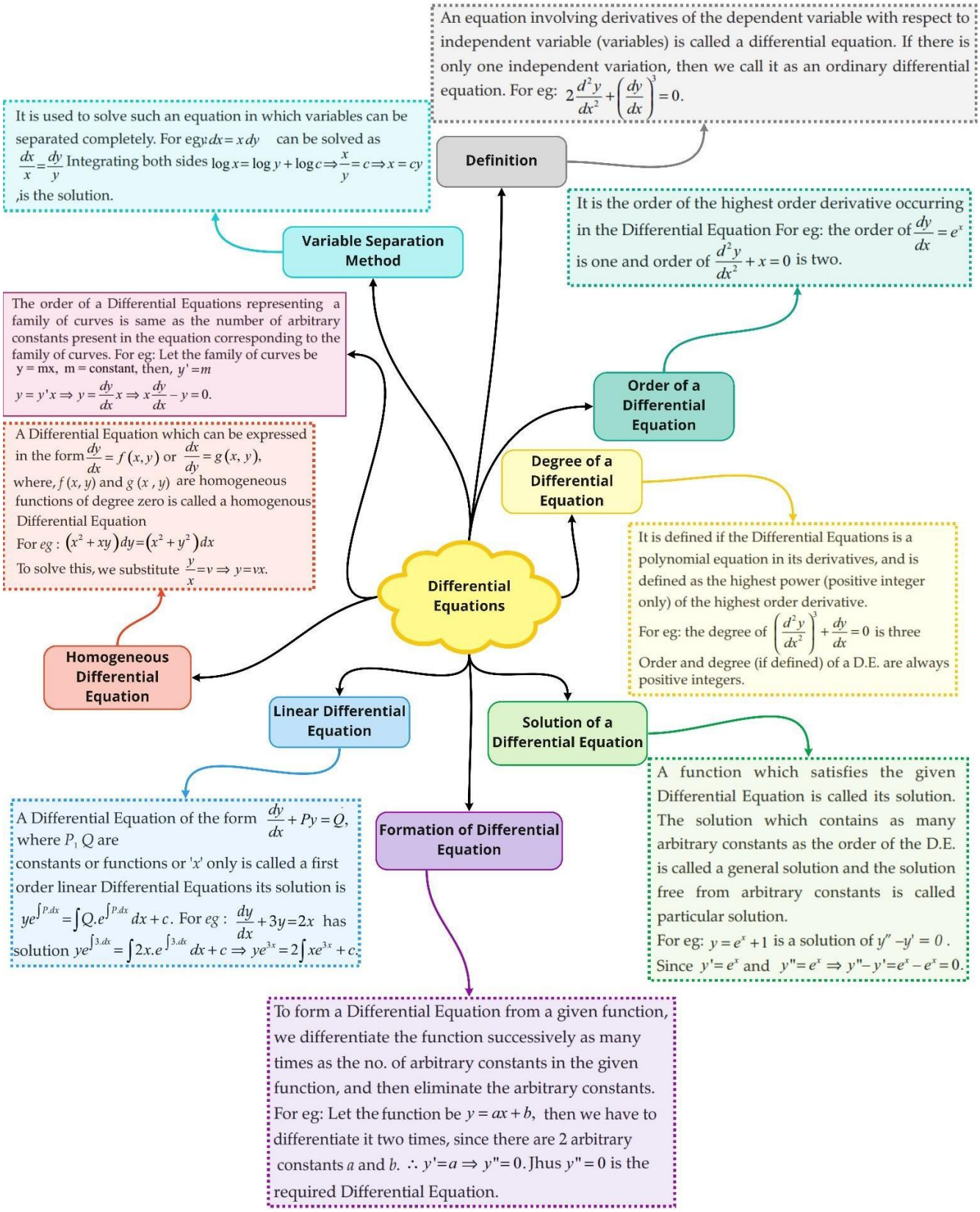
MATHEMATICS

(DIFFERENTIAL EQUATIONS)

DIFFERENTIAL EQUATIONS

1. **Differential Equation:** An equation involving derivatives of the dependent variable with respect to independent variable (variables) is known as a differential equation.
2. **Linear and non-linear differential equation:** A differential equation is said to be linear if unknown function (dependent variable) as its derivative which occurs in the equation, occur only in the first degree, and are not multiplied together. Otherwise, the differential equation is said to be non-linear.
3. **Order:** Order of a differential equation is the order of the highest order derivative occurring in the differential equation.
4. **Degree:** Degree of a differential equation is defined if it is a polynomial equation in its derivatives.
5. Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.
6. **Solution:** A function which satisfies the given differential equation is called its solution.
7. **General Solution:** The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution.
8. **Particular Solution:** The solution free from arbitrary constants is called particular solution.
9. To form a differential equation from a given function we differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.
10. **Variable Separable method:** Variable separable method is used to solve such an equation in which variables can be separated completely i.e., terms containing y should remain with dy and terms containing x should remain with dx.
11. A differential equation which can be expressed in the form $\frac{dy}{dx} f(x,y)$ or $\frac{dx}{dy} g(x,y)$ where, f(x, y) and g (x, y) are homogenous functions of degree zero is called a homogeneous differential equation.
12. A differential equation of the form $\frac{dy}{dx} + Py$ = Q, where P and Q are constants or functions of x only is called a first order linear differential equation.

Class : 12th Maths
Chapter- 9 : Differential Equations



Important Questions

Multiple Choice questions-

1. The degree of the differential equation:

$(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^2 + \sin(\frac{dy}{dx}) + 1 = 0$ is

(a) 3

(b) 2

(c) 1

(d) not defined.

2. The order of the differential equation:

$2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is

(a) 2

(b) 1

(c) 0

(d) not defined.

3. The number of arbitrary constants in the general solution of a differential equation of fourth order is:

(a) 0

(b) 2

(c) 3

(d) 4.

4. The number of arbitrary constants in the particular solution of a differential equation of third order is:

(a) 3

(b) 2

(c) 1

(d) 0.

5. Which of the following differential equations has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + 1 = 0$

(d) $\frac{d^2y}{dx^2} - 1 = 0$

6. Which of the following differential equations has $y = x$ as one of its particular solutions?

(a) $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$

(b) $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = x$

(c) $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$

(d) $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = 0$

7. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(a) $e^x + e^{-y} = c$

(b) $e^x + e^y = c$

(c) $e^{-x} + e^y = c$

(d) $e^{-x} + e^{-y} = c$.

8. Which of the following differential equations cannot be solved, using variable separable method?

(a) $\frac{dy}{dx} + e^{x+y} + e^{-x+y}$

(b) $(y^2 - 2xy)\,dx = (x^2 - 2xy)\,dy$

(c) $xy\frac{dy}{dx} = 1 + x + y + xy$

(d) $\frac{dy}{dx} + y = 2$.

9. A homogeneous differential equation of the form $\frac{dy}{dx} = h(\frac{x}{y})$ can be solved by making the substitution.

(a) y = vx

(b) v = yx

(c) x = vy

(d) x = v

10. Which of the following is a homogeneous differential equation?

(a) $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

(b) $xy\,dx - (x^3 + y^2)dy = Q$

(c) $(x^3 + 2y^2)\,dx + 2xy\,dy = 0$

(d) $y^2\,dx + (x^2 - xy - y^2)dy = 0$.

Very Short Questions:

1. Find the order and the degree of the differential equation: $x^2\frac{d^2y}{dx^2} = [1 + (\frac{dy}{dx})^2]^4$

2. Determine the order and the degree of the differential equation: $(\frac{dy}{dx})^3 + 2y\frac{d^2y}{dx^2} = 0$

3. Form the differential equation representing the family of curves: y = b (x + a), where « and b are arbitrary constants.

4. Write the general solution of differential equation:

   $\frac{dy}{dx} = e^{x+y}$

5. Find the integrating factor of the differential equation:

   $y\frac{dy}{dx} - 2x = y^3e^{-y}$

6. Form the differential equation representing the family of curves y = a sin (3x – b), where a and b are arbitrary constants.

Short Questions:

1. Determine the order and the degree of the differential equation:

2. Form the differential equation representing the family of curves: $y = e^{2x}(a + bx)$, where 'a' and 'h' are arbitrary constants.

3. Solve the following differentia equation:

   $\frac{dy}{dx} + y = \cos x - \sin x$

4. Solve the following differential equation:

   $\frac{dx}{dy} + x = (\tan y + \sec 2y)$

Long Questions:

1. Find the area enclosed by the circle:

   $x^2 + y^2 = a^2$.

2. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$.

3. Find the area bounded by the curves $y = \sqrt{x}$, 2y + 3 = Y and Y-axis.

4. Find the area of region:

   $\{(x,y): x^2 + y^2 < 8, x^2 < 2y\}$.

Case Study Questions:

1. If the equation is of the form $\frac{dy}{dx} = py = Q,$ where P, Q are functions of x, then the solution of the differential equation is given by $ye^{\int pdx} = \int Q\, e^{\int pdx} dx + c,$ where $e^{\int pdx}$ is called the integrating factor (I.F.).

Based on the above information, answer the following questions.

i. The integrating factor of the differential equation

$\sin x\frac{dy}{dx} + 2y\cos x = 1$ is $(\sin x)^{\lambda}$, where $\lambda =$

a. 0
b. 1
c. 2
d. 3

ii. Integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} - xy = 1$ is:

a. $-x$

b. $\frac{x}{1+x^2}$

c. $\sqrt{1-x^2}$

d. $\frac{1}{2}\log(1-x^2)$

iii. The solution of $\frac{dy}{dx} + y = e^{-x},\ y(0) = 0,$ is:

a. $y = e^{x}(x-1)$

b. $y = xe^{-x}$

c. $y = xe^{-x} + 1$

d. $y = (x+1)e^{-x}$

iv. General solution of $\frac{dy}{dx} + y\tan x = \sec x$ is:

a. $y\sec y = \tan x + c$

b. $y\tan x = \sec x + c$

c. $\tan x = y\tan x + c$

d. $x\sec x = \tan y + c$

v. The integrating factor of differential equation $\frac{dy}{dx} - 3y = \sin 2x$ is:

a. e^{3x}

b. e^{-2x}

c. e^{-3x}

d. xe^{-3x}