NOTES

– NOTES WITH MIND MAPS –

MATHEMATICS

(APPLICATION OF DERIVATIVES)

12TH

APPLICATION OF DERIVATIVES

1. If aquantity y varies with another quantity x, satisfying some rule y = f(x), then $\frac{dx}{dy}$ $(or\ f\ '(x)$ represents the rate of change of y with respect to x and $\frac{dy}{dx}\Big]_{x=x_0}$ $(or\ f'(x_0)$ represents the rate of change of y with respect to x at $x = x_o$.

2. If two variables x and y are varying with respect to another variable t, i.e., if x = f(t) and y =g(t) then by Chain Rule
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \text{ if } \frac{dx}{dt} \neq 0$$

3. A function f is said to be increasing on an interval (a, b) if $x_1 < x_2$ in (a, b) ⇒ $f(x_1) < f(x_2)$ for all x_1, $x_2 \in$ (a, b). Alternatively, if f'(x) > 0 for each x in, then f(x) is an increasing function on (a, b).

4. A function f is said to be increasing on an interval (a, b) if $x_1 < x_2$ in (a, b) ⇒ $f(x_1) > f(x_2)$ for all x_1, $x_2 \in$ (a, b). Alternatively, if f'(x) > 0 for each x in, then f(x) is an decreasing function on (a, b).

5. The equation of the tangent at (x_0, y_0) to the curve y = f (x) is given by
$$y - y_o = \frac{dy}{dx}\Big]_{(x_0, y_0)}(x - x_o)$$

6. If $\frac{dy}{dx}$ does not exist at the point (x_0, y_0), then the tangent at this point is parallel to the y-axis and its equation is $x = x_0$.

7. If tangent to a curve y = f(x) at $x = x_0$ is parallel to x-axis, then $\frac{dy}{dx}\Big]_{x=x_0} = 0$

8. **Equation of the normal** to the curve y = f (x) at a point (x_o, y_o), is given by
$$y - y_o = \frac{-1}{\frac{dy}{dx}\Big]_{(x_0, y_0)}}(x - x_0)$$

9. If $\frac{dy}{dx}$ at the point (x_o, y_o), is zero, then equation of the normal is $x = x_0$.

10. If $\frac{dy}{dx}$ at the point (x_o, y_o), does not exist, then the normal is parallel to x-axis and its equation is $y = y_0$.

11. Let y = f (x), Δx be a small increment in x and Δy be the increment in y corresponding to the increment in x, i.e., Δy = f (x + Δx) – f (x). Then dy given by dy = f'(x) dx or $dy = (\frac{dy}{dx})\, dx$ is a good of Δy when dx x = Δ is relatively small and we denote it by dy ≈ Δy.

12. A point c in the domain of a function f at which either f '(c) = 0 or f is not differentiable is called a critical point of f.

13. First Derivative Test: Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then,

i. If f '(x) changes sign from positive to negative as x increases through c, i.e., if f '(x) > 0 at every point sufficiently close to and to the left of c, and f '(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of local maxima.

ii. If f '(x) changes sign from negative to positive as x increases through c, i.e., if f '(x) < 0 at every point sufficiently close to and to the left of c, and f '(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of local minima.

iii. If f '(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. In fact, such a point is called point of inflexion.

14. Second Derivative Test: Let f be a function defined on an interval I and c ∈ I. Let f be twice differentiable at c. Then,

i. x = c is a point of local maxima if f '(c) = 0 and f "(c) < 0

The values f (c) is local maximum value of f.

ii. x = c is a point of local minima if f '(c) = 0 and f "(c) > 0

In this case, f (c) is local minimum value of f.

iii. The test fails if f '(c) = 0 and f "(c) = 0.

In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a point of inflexion.

15. Working rule for finding absolute maxima and/ or absolute minima

Step 1: Find all critical points of f in the interval, i.e., find points x where either f '(x) = 0 or f is not differentiable.

Step 2: Take the end points of the interval.

Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify the maximum and minimum values of f out of the values calculated in Step3.

This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

Class : 12th Maths
Chapter- 6 : Applications of Derivatives

Applications of Derivatives

Approximations

Let $y=f(x)$ Δx be a small increment in 'x' and Δy be the small increment in y corresponding to the increment in 'x', i.e. $\Delta y = f(x+\Delta x) - f(x)$. Then, Δy is given by $dy = f'(x)dx$ or $dy = \left(\frac{dy}{dx}\right)\Delta x$, is a good approximation of Δy when $dx=\Delta x$ is relatively small and denote by $dy \approx \Delta y$. For eg: Let us approximate $\sqrt{36.6}$. To do this, we take $y=\sqrt{x}, x = 36, \Delta x = 0.6$ then $\Delta y = \sqrt{x+dx} - \sqrt{x}$

$= \sqrt{36.6} - \sqrt{36}$

$= \sqrt{36.6} - 6 \Rightarrow \sqrt{36.6} = 6 + \bar{d}y$

Now, dy is approximately Δy and is given by $\bar{d}y$

$= \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\sqrt{x}}(0.6) = \frac{1}{2\sqrt{36}}(0.6) = 0.05.$ So, $\sqrt{36.6} \approx 6 + 0.05 = 6.05.$

Rate of Change Quantities

If a quantity if 'y' varies with another quantity x so that $y = f(x)$, then $\frac{dy}{dx}\left[f'(x)\right]$ represents the rate of change of y w.r.t x and $\left.\frac{dy}{dx}\right|_{x=x_o} \left(f'(x_0)\right)$ represents the rate of change of y w.r.t. x at $x = x_o$

If 'x' and 'y' varies with another variable 't' i.e., if $x = f(t)$ and $y = g(t)$, then by chain rule $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$, if $\frac{dx}{dt} \neq 0$.

For eg: if the radius of a circle, r = 5 cm, then the rate of change of the area of a circle per second w.r.t 'r' is –

$\frac{da}{dr}|_{r=5} = \frac{d}{dr}(\pi r^2)|_{r=5} = 2\pi r|_{r=5} = 10\pi$

Maximum and Minima

A point C in the domain of 'f' at which either $f'(c)=0$ or is not differentiable is called a critical point of f.

Second Derivative test

Let f be a function defined on I and CC–I, f is twice differentiable at C. Then **(i)** $x=C$ is a point of local max. If $f'(C)=0$ and $f''(C)<0$, $f(C)$ is local max. of f. **(ii)** $x=C$ is a point of local min if $f'(C)=0$ and $f''(C)>0$. $f(C)$ is local min of f. **(iii)** The test fails if $f'(C)=0$ and $f''(C)=0$

First Derivative test

Let f be continuous at a critical point C in open I. Then **(i)** if $f'(x) > 0$ at every point left of C and $f'(x) < 0$ at every point right of C, then 'C' is a point of local maxima. **(ii)** If $f'(x) < 0$ at every point left of C and $f'(x) > 0$ at every point right of C, then 'C' is a point of local minima.
(iii) If $f'(x)$ does not change sign as 'x' increases through C, then 'C' is called the point of inflection.

Increasing and decreasing functions

A function f is said to be (i) increasing on (a,b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \leq f(x_2) \forall x_1, x_2 \in (a,b)$, and (ii) decreasing on (a, b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) > f(x_2) \forall x_1 x_2 \in (a,b)$

If $f'(x) \geq 0 \forall x \in (a,b)$ then f is increasing in (a,b) and if $f'(x) \leq 0 \forall x \in (a,b)$, then f is decreasing in (a,b) For eg: Let $f(x) = x^3 - 3x^2 + 4x, x \in R$, then $f'(x) = 3x^2 - 6x + 4 = 3(x-1)^2 + 1 > 0 \forall x \in R$. So, the function f is strictly increasing on R.

Tangents and Normals

The equation of the tangent at (x_0, y_0), to the curve $y = f(x)$ is given by $(y - y_0) = \left.\frac{dy}{dx}\right|_{(x_0, y_0)}(x - x_0)$ if $\frac{dy}{dx}$ does not exists at (x_0, y_0), then the tangent at (x_0, y_0) is parallel to the y–axis and its equation is $x = x_0$. If tangent to a curve $y = f(x)$ at $x = x_0$ is parallel to x–axis, then $\frac{dy}{dx}|_{x=x_0} = 0$.

Equation of the normal to the curve

$y = f(x)$ at (x_0, y_0) is $y - y_0 = -\frac{1}{\left.\frac{dy}{dx}\right|_{(x_0, y_0)}}(x - x_0)$ if $\frac{dy}{dx}$ at (x_0, y_0) is zero, then equation of the normal is $x = x_0$. If $\frac{dy}{dx}$ at (x_0, y_0) does not exist, then the normal is parallel to x-axis and its equation is $y = y_0$. For eg: Let $y = x^3 - x$ be a curve, then the slope of the tangent to $y = x^3 - x$ at $x = 2$ is $\left.\frac{dy}{dx}\right|_{x=2} = 3x^2 - 1 = 3.2^2 - 1 = 11$

Case Study Questions:

1. An architecture design a auditorium for a school for its cultural activities. The floor of the auditorium is rectangular in shape and has a fixed perimeter P.

Based on the above information, answer the following questions.

i. If x and y represents the length and breadth of the rectangular region, then relation between the variable is.

a. $x + y = P$
b. $x^2 + y^2 = P^2$
c. $2(x + y) = P$
d. $x + 2y = P$

ii. The area (A) of the rectangular region, as a function of x, can be expressed as.

a. $A = px + \frac{x}{2}$

b. $A = \frac{px+x^2}{2}$

c. $A = \frac{px-2x^2}{2}$

d. $A = \frac{x^2}{2} + px^2$

iii. School's manager is interested in maximising the area of floor 'A' for this to be happen, the value of x should be.

a. P

b. $\frac{P}{2}$

c. $\frac{P}{3}$

d. $\frac{P}{4}$

iv. The value of y, for which the area of floor is maximum, is.

a. $\frac{P}{2}$

b. $\frac{P}{3}$

c. $\frac{P}{4}$

d. $\frac{P}{16}$

v. Maximum area of floor is.

a. $\frac{P^2}{16}$

b. $\frac{P^2}{64}$

c. $\frac{P^2}{4}$

d. $\frac{P^2}{28}$