

- NOTES WITH MIND MAPS -

MATHEMATICS

(APPLICATION OF DERIVATIVES)



Class 12th Mathematics

APPLICATION OF DERIVATIVES

1. If aquantity y varies with another quantity x, satisfying some rule y = f(x), then

 $\frac{dx}{dy}$ (or f'(x) represents the rate of change of y with respect to x and $\frac{dy}{dx} = x_0$ (or $f'(x_0)$)

represents the rate of change of y with respect to x at $x = x_0$.

If two variables x and y are varying with respect to another variable t, i.e., if x = f(t) and y =g(t) then by Chain Rule

$$\frac{dy}{dx} = \frac{dy}{dt}, \text{ if } \frac{dx}{dt} \neq 0$$

- **3.** A function f is said to be increasing on an interval (a, b) if $x_1 < x_2$ in (a, b) \Rightarrow f(x₁) < f(x₂) for all x₁, $x_2 \in (a, b)$. Alternatively, if f'(x) > 0 for each x in, then f(x) is an increasing function on (a, b).
- **4.** A function f is said to be increasing on an interval (a, b) if $x_1 < x_2$ in (a, b) \Rightarrow f(x₁) > f(x₂) for all x₁, $x_2 \in (a, b)$. Alternatively, if f'(x) > 0 for each x in, then f(x) is an decreasing function on (a, b).
- 5. The equation of the tangent at (x_0, y_0) to the curve y = f(x) is given by

$$y - y_o = \frac{dy}{dx} \Big|_{(x_0, y_0)} (x - x_o)$$

- 6. If $\frac{dy}{dx}$ does not exist at the point (x₀, y₀), then the tangent at this point is parallel to the y-axis and its equation is x = x₀.
- 7. If tangent to a curve y = f(x) at $x = x_0$ is parallel to x-axis, then $\frac{dy}{dx} = 0$
- 8. Equation of the normal to the curve y = f(x) at a point (x_0, y_0) , is given by

$$y - y_{o} = \frac{-1}{\frac{dy}{dx}} (x - x_{0})$$

- **9.** If $\frac{dy}{dx}$ at the point (x₀, y₀), is zero, then equation of the normal is x = x₀.
- **10.** If $\frac{dy}{dx}$ at the point (x_o, y_o), does not exist, then the normal is parallel to x-axis and its equation is $y = y_0$.

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- **11.** Let y = f(x), Δx be a small increment in x and Δy be the increment in y corresponding to the increment in x, i.e., $\Delta y = f(x + \Delta x) f(x)$. Then dy given by dy = f'(x) dx or $dy = (\frac{dy}{dx}) dx$ is a good of Δy when $dx x = \Delta$ is relatively small and we denote it by $dy \approx \Delta y$.
- 12. A point c in the domain of a function f at which either f '(c) = 0 or f is not differentiable is called a critical point of f.
- 13. First Derivative Test: Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then,
 - i. If f'(x) changes sign from positive to negative as x increases through c, i.e., if f'(x) > 0 at every point sufficiently close to and to the left of c, and f'(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of local maxima.
 - ii. If f'(x) changes sign from negative to positive as x increases through c, i.e., if f'(x) < 0 at every point sufficiently close to and to the left of c, and f'(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of local minima.
 - iii. If f'(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. In fact, such a point is called point of inflexion.
- **14. Second Derivative Test:** Let f be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c. Then,
 - i. x = c is a point of local maxima if f'(c) = 0 and f''(c) < 0

The values f (c) is local maximum value of f.

ii. x = c is a point of local minima if f'(c) = 0 and f''(c) > 0

In this case, f (c) is local minimum value of f.

iii. The test fails if f'(c) = 0 and f''(c) = 0.

In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a point of inflexion.

15. Working rule for finding absolute maxima and/ or absolute minima

Step 1: Find all critical points of f in the interval, i.e., find points x where either f '(x) = 0 or f is not differentiable.

Step 2: Take the end points of the interval.

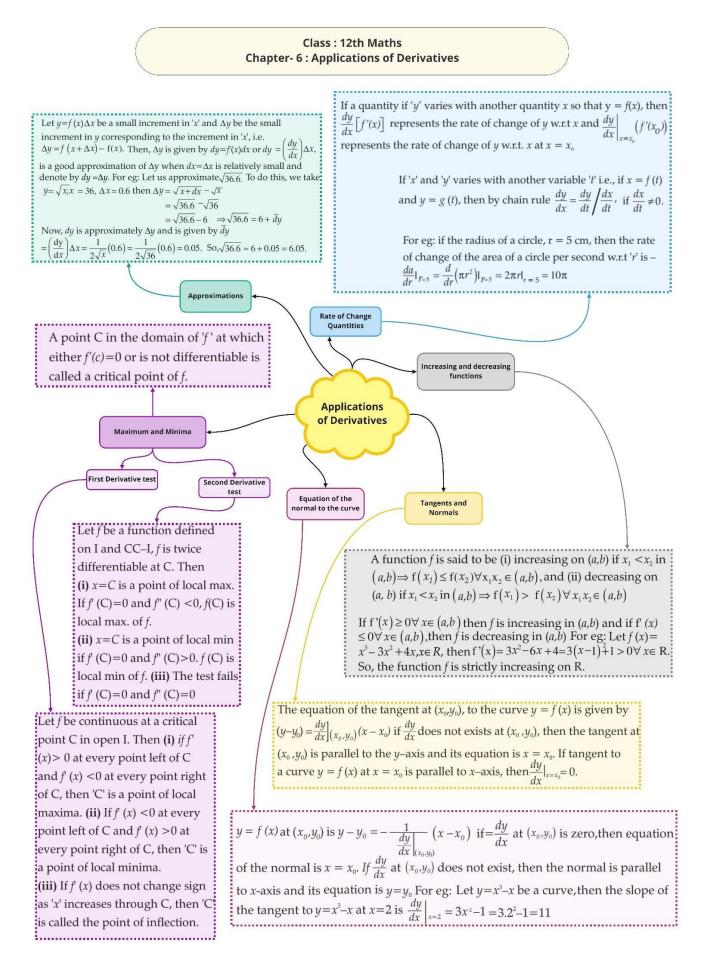
Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify the maximum and minimum values of f out of the values calculated in Step3.

This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

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Case Study Questions:

1. An architecture design a auditorium for a school for its cultural activities. The floor of the auditorium is rectangular in shape and has a fixed perimeter P.



Based on the above information, answer the following questions.

i. If x and y represents the length and breadth of the rectangular

region, then relation between the variable is.

a. x + y = Pb. $x^2 + y^2 = P^2$ c. 2(x + y) = Pd. x + 2y = P

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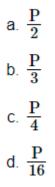
- ii. The area (A) of the rectangular region, as a function of x, can be expressed as.
 - a. A = px + $\frac{x}{2}$ b. A = $\frac{px+x^2}{2}$ c. A = $\frac{px-2x^2}{2}$ d. A = $\frac{x^2}{2} + px^2$

iii. School's manager is interested in maximising the area of floor

'A' for this to be happen, the value of x should be.

a. **P** b. $\frac{P}{2}$ c. $\frac{P}{3}$ d. $\frac{P}{4}$

iv. The value of y, for which the area of floor is maximum, is.



V. Maximum area of floor is.

- a. $\frac{P^2}{16}$ b. $\frac{P^2}{64}$ c. $\frac{P^2}{4}$
- d. $\frac{P^2}{28}$