NOTES

– NOTES WITH MIND MAPS –

MATHEMATICS

(INVERSE TRIGNOMETRIC FUNCTIONS)

12TH

INVERSE TRIGONOMETRIC FUNCTIONS

1. The domains and ranges (principal value branches) of inverse trigonometric functions are given in the following table:

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1}x$	[-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	[-1, 1]	$[0, \pi]$
$y = \operatorname{cosec}^{-1}x$	R- [-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}x$	R-[-1, 1]	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \tan^{-1}x$	R	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cot^{-1}x$	R	$[0, \pi]$

2. $\sin^{-1}x$ should not be confused with ($\sin^{-1}x$). In fact, $(\sin x)^{-1} = \frac{1}{sinsinx}$ And similarly for other trigonometric functions.

3. The value of an inverse trigonometric functions which lies in its principal value branch is called the principal value of that inverse trigonometric functions.

4. **For suitable values of domain, we have**

- $y = \sin^{-1}x \Rightarrow x = \sin y$

- $x = \sin y \Rightarrow y = \sin^{-1} x$
- $\sin(\sin^{-1} x) = x$
- $\sin^{-1}(\sin x) = x$
- $\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1} x$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x$
- $\cos^{-1}\frac{1}{x} = \sec^{-1} x$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x$
- $\tan^{-1}\frac{1}{x} = \cot^{-1} x$

  $\cot^{-1}\frac{1}{x} = \tan^{-1} x$

  $\operatorname{cosec}^{-1}\frac{1}{x} = \sin^{-1} x$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x$

  $\sec^{-1}\frac{1}{x} = \cos^{-1} x$
- $\sin^{-1}(-x) = -\sin^{-1} x$

- $\tan^{-1}(-x) = -\tan^{-1} x$
- $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
- $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$
- $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$$\cos^{-1}x + o\cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

- $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$
- $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$2\cos^{-1}x = \cos^{-1}\left(2x^2-1\right)$$

- $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-\sqrt{x^2}}\right)$

  $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$

  $3\cos^{-1}x = \cos^{-1}(4x^3-3x)$

  $3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

5. **Conversion:**

- $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \cos ec^{-1}\frac{1}{x}$
- $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\frac{1}{x} = \cos ec^{-1}\frac{1}{\sqrt{1-x^2}}$
- $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \sec^{-1}\sqrt{1+x^2} = \cos ec^{-1}\frac{\sqrt{1+x^2}}{x} = \cot^{-1}\frac{1}{x}$
- $\cot^{-1}x = \sin^{-1}\frac{1}{\sqrt{1+x^2}} = \cos^{-1}\frac{x}{\sqrt{1+x^2}} = \sec^{-1}\frac{1}{x} = \sec^{-1}\frac{\sqrt{1+x^2}}{x} = \cos ec^{-1}\sqrt{1+x^2}$
- $\sec^{-1}x = \tan^{-1}\frac{\sqrt{x^2-1}}{1} = \cot^{-1}\frac{1}{\sqrt{x^2-1}} = \sin^{-1}\frac{\sqrt{x^2-1}}{x} = \cos^{-1}\frac{1}{x} = \cos ec^{-1}\frac{x}{\sqrt{x^2-1}}$
- $\cos ec^{-1}x = \sin^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{\sqrt{x^2-1}} = \cot^{-1}\sqrt{x^2-1} = \sec^{-1}\frac{x}{\sqrt{x^2-1}} = \cos^{-1}\frac{\sqrt{x^2-1}}{x}$

6. **Some other properties of Inverse Trigonometric Function:**

- $\tan^{-1}\dfrac{x}{\sqrt{a^2-x^2}}=\sin^{-1}\dfrac{x}{a}$
- $\cot^{-1}\dfrac{x}{\sqrt{a^2-x^2}}=\cos^{-1}\dfrac{x}{a}$
- $\tan^{-1}\dfrac{a}{\sqrt{x^2-a^2}}=\operatorname{cosec}^{-1}\dfrac{x}{a}$
- $\cot^{-1}\dfrac{a}{\sqrt{x^2-a^2}}=\sec^{-1}\dfrac{x}{a}$