NOTES

- NOTES WITH MIND MAPS -

# MATHEMATICS

## (SEQUENCES AND SERIES)

11TH

# SEQUENCE AND SERIES

## Top Definitions

1. A Sequence is an ordered list of numbers and has the same meaning as conversational english. Asequence is denoted by $<a_n>_{n \geq 1} = a_1, a_2, a_3, \ldots\ldots a_n$.
2. The various numbers occurring in a sequence are called its terms.
3. A sequence containing finite number of terms is called a finite sequence. A finite sequence has the last term.
4. A sequence which is not a finite sequence, i.e., containing an infinite number of terms is called aninfinite sequence. There is no last term in an infinite sequence.
5. A sequence is said to be an arithmetic progression if every term differs from the preceding term by a constant number. For example, the sequence $a_1, a_2, a_3, \ldots a_n$ is called an arithmetic sequence or AP if $a_{n+1} = a_n + d$.

   For all $n \in N$, where d is a constant called the common difference of AP.
6. A is the arithmetic mean of two numbers a and b if a, A and b forms AP.
7. A sequence is said to be a geometric progression or GP if the ratio of any tem to its preceding term isthe same throughout. Constant ratio is a common ratio denoted by r.
8. If three numbers are in GP, then the middle term is called the geometric mean of the other two.

## Top Concepts

1. A sequence has a definite first member, second member, third member and so on.
2. The nth term $<a_n>$ is called the general term of the sequence
3. Fibonacci sequence 1, 1, 2, 3, 5, 8, .. is generated by a recurrence relation given by

   $a_1 = a_2 = 1.$

   $a_3 = a_1 + a_2 \ldots\ldots$

   $a_n = a_{n-2} + a_{n-1}, n > 2.$

4. A sequence is a function with a domain, the set of natural numbers or any of its subsets of the type {1, 2, 3, ... k}.
5. If the number of terms is three with a common difference 'd', then the terms are a- d, d and a+ d
6. If the number of terms is four with common difference '2d', then the terms are a-3d, a- d, a + d and a + 3d.
7. If the number of terms is five with a common difference 'd', then the terms are a - 2d, a - d, a, a + d and a + 2d.
8. If the number of terms is six with a common difference '2d', then the terms are a - 5d, a - 3d, a - d, a + d, a + 3d and a + 5d.
9. The sum of the series is the number obtained by adding the terms.
10. The general form of AP is a, a + d, a + 2d, ...a + (n - 1) d. The term a is called the **first term** of AP and d is called the **common difference** of AP. The term d can be any real number.
11. If d>0, then AP is increasing, if d < 0 then AP is decreasing, and if d = 0, then AP is constant.
12. For AP, a, (a + d), (a + 2d), ..., (λ − 2d), (λ − d), λ with the first term a and common difference d and last term λ and general term is λ − (n − 1)d.
13. Let a be the first term and d be the common difference of AP with 'm' terms. Then nth term from theend is the $(m- n + 1)^{th}$ term from the beginning.
14. Properties of AP
    i. If a constant is added to each term of AP, then the resulting sequence is also AP.
    ii. If a constant is subtracted from each term of AP, then the resulting sequence is also AP.
    iii. If each term of AP is multiplied by a constant, then the resulting sequence is also an AP.
    iv. If each term of AP is divided by a non-zero constant, then the resulting sequence is also AP.
15. The arithmetic mean A of any two numbers a and b is given by $\frac{a+b}{2}$.
16. General form of GP: a, ar, $ar^2$, $ar^3$,....., where a is the first term and r is the constant ratio. The term r can take any non-zero real number.

17. A sequence in GP will remain in GP if each of its terms is multiplied by a non-zero constant.

18. A sequence obtained by multiplying the two GPs term by term results in GP with common ratio theproduct of the common ratio of the two GPs.

19. The reciprocals of the terms of a given GP form a GP with common ratio $\frac{1}{r}$.

20. If each term of GP is raised to the same power, then the resulting sequence also forms GP.

21. The geometric mean (GM) of any two positive numbers a and b is given by $\sqrt{ab}$.

22. Let A and G be AM and GM of two given positive real numbers a and b, respectively, then A ≥ G.

    Where $A = \frac{a+b}{2}$ and G = $\sqrt{ab}$

23. Let A and G be AM and GM of two given positive real numbers a and b, respectively. Then, the quadratic equation having a and b as its roots is $x_2$ - 2Ax + G = 0.

24. Let A and G be AM and GM of two given positive real numbers a and b, respectively. Then the given numbers are $A \pm \sqrt{A^2 - G}$.

25. If AM and GM are in the ratio m:n, then the given numbers are in the ratio

    $m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$.

## Top Formulae

1. The n[th] term or general term of AP is $a_n = a + (n - 1)d$, where a is the first term and d is the commondifference.

2. The general term of AP given its last term is $l - (n - 1)d$.

3. A sequence is AP if and only if its nth term is a linear expression in n,
   $A_n$ = Xn+ Y, where X and Y are constants.

4. Let a, a + d, a + 2d,.., a + (n − 1)d be AP. Then

   $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + \ell]$ where $\lambda = a + (n - 1)d$

5. A sequence is AP if and only if the sum of its nth term is an expression of the form $X_n^2$ + Y, where X and Y are constants.

6. If the terms of an AP are selected in regular intervals, then the selected terms form an AP.

Let $a_n$, $a_{n+1}$ and $a_{n+2}$ be the consecutive terms of AP.

Then $2a_{n+1} + = a_n\, a_{n+2}$.

7. Let $A_1, A_2, A_3, \ldots A_n$ be n numbers between a and b such that $a, A_1, A_2, A_3, \ldots A_n, b$ is AP. The n numbers between a and b are as follows

$$A_1 = a + d = a + \frac{b-a}{n+1}$$

$$A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$$

$$A_3 = a + 3d = a + \frac{3(b-a)}{n+1}$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$A_n = a + nd = a + \frac{n(b-a)}{n+1}$$

8. General term of GP is $ar^{n-1}$, where a is the first term and r is the common ratio.

9. If the number of terms is 3 with the common ratio r, then the selection of terms should be $\frac{a}{r}$, and ar.

10. If the number of terms is 4 with the common ratio r2 , then the selection of terms should be $\frac{a}{r^3}$, a ar and $ar^2$.

11. If the number of terms is 5 with the common ratio r, then the selection of terms should be $\frac{a}{r^2}$, a, a, ar and $ar^2$.

12. Sum to the first n terms of GP $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

(i) If r = 1, $S_n = a + a + a + \ldots + a$ (n terms) = na.

(ii) If r<1, $S_n = \frac{q(1-r^n)}{1-r}$.

(iii)If r>1, $S_n = \frac{a(r^n - 1)}{r-1}$.

13. Let $G_1, G_2, \ldots\ldots, G_n$ be n numbers between positive numbers a and b such that $a, G_1, G_2, G_3, \ldots\ldots G_n, b$ is GP.

Thus $b = br^{n+1}$, or $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, $G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$, $G_3\ ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$

$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

14. The sum of infinite GP with the first term a and common ratio r ($-1 < r < 1$) is $S = \frac{a}{1-r}$

## Some Special Series

1. The sum of first n natural numbers is

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

2. Sum of squares of the first n natural numbers

$$1^2+2^2+3^2+\ldots\ldots..n^2= \frac{n(n+1)(2n+1)}{6}$$

3. Sum of cubes of first n natural numbers

$$1^3 + 2^3 + \ldots + n^3= \frac{n^2(n+1)^2}{4} = \frac{[n(n+1)]^2}{}$$

4. Sum of powers of 4 of first n natural numbers

$$1^4+2^4+3^4+\ldots.+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

5. Consider the series $a_1 + a_2 + a_3 + a_4 + \ldots + an + \ldots.$

   If the differences in $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$, …. are in AP, then the nth term is given by $a_n = an^2 + bn + c$, where a, b and c are constants.

6. Consider the series $a_1 + a_2 + a_3 + a_4 + \ldots + an + \ldots.$

   If the differences in $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$, …. are in GP then the nth term is given by

   $an = ar^{n-1} + b + c$, where a, b and c are constants.