NOTES

- NOTES WITH MIND MAPS -

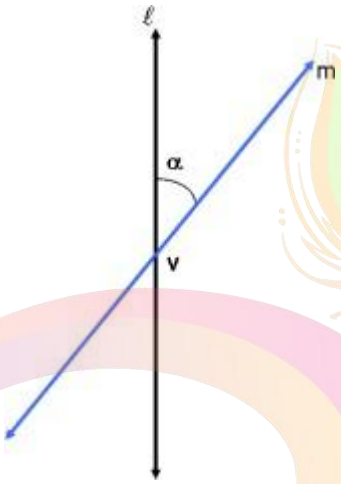
# MATHEMATICS

## (CONIC SECTIONS)

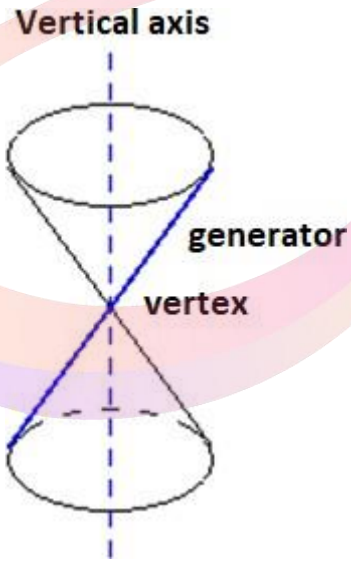
# CONIC SECTIONS

## Key Concepts

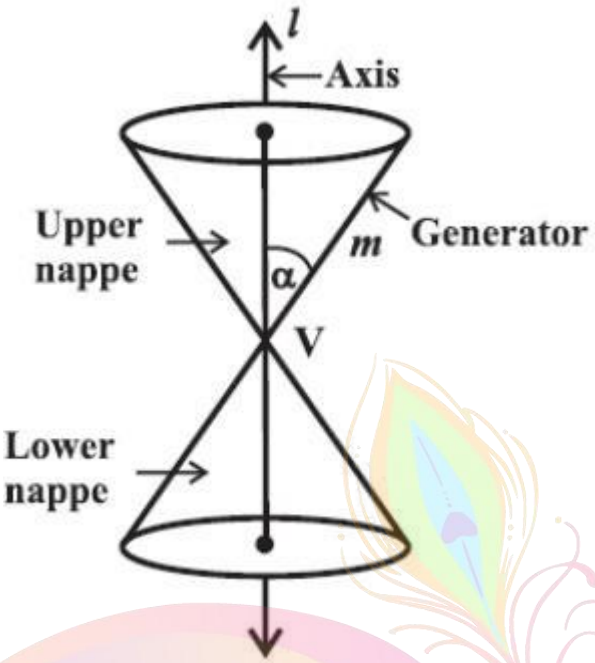
1. Let λ be a fixed vertical line and m be another line intersecting it at a fixed point V and inclined to it atan angle α.




On rotating the line m around the line λ in such a way that the angle α remains constant, the surface generated is a double-napped right-circular hollow cone.




2. The point V is called the vertex; the line λ is the axis of the cone. The rotating line m is called agenerator of the cone. The vertex separates the cone into two parts called nappes.




3. The sections obtained by cutting a double napped cone with a plane are called conic sections. Conicsections are of two types (i) degenerate conics (ii) non-degenerate conics.

4. If the cone is cut at its vertex by the plane, then degenerate conics are obtained

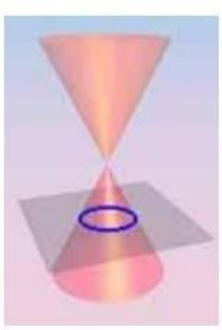
5. If the cone is cut at the nappes by the plane, then non-degenerate conics are obtained.

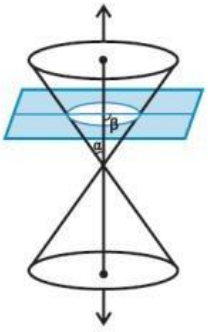
6. Degenerate conics are points, lines and double lines.

7. Circle, parabola, ellipse and hyperbola are degenerate conics.

8. When the plane cuts the nappes (other than the vertex) of the cone, degenerate conics are obtained.

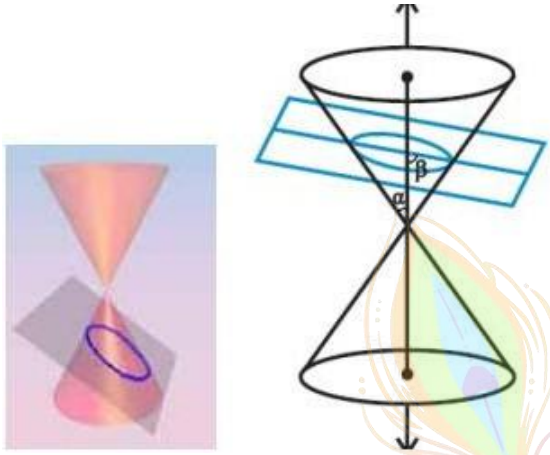
   (a) When $\beta = 90°$ , the section is a circle.





The plane cuts the cone horizontally.

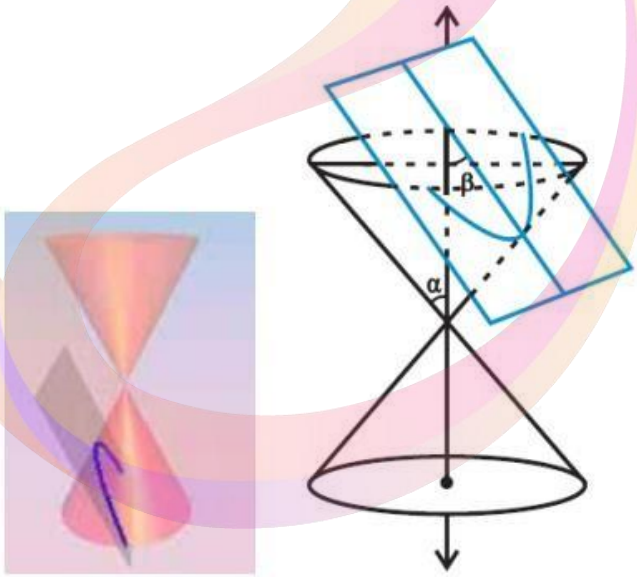
(b) When $\alpha < \beta < 90^{o}$, the section is an ellipse.




**Ellipse**

The plane cuts one part of the cone in an inclined manner.

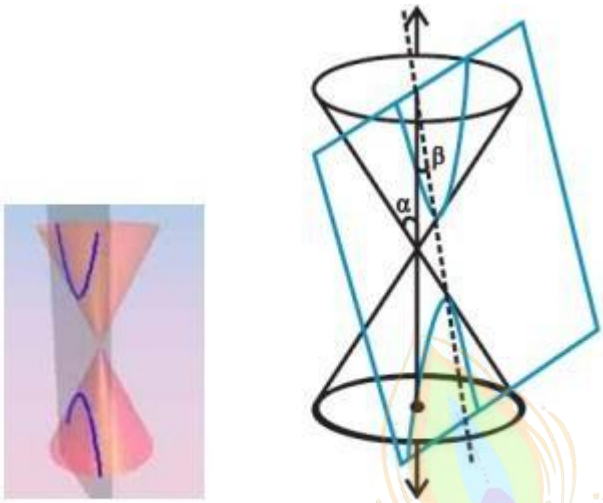
(c) When $\beta = \alpha$, the section is a parabola.




**Parabola**

The plane cuts the cone in such a way that it is parallel to a generator.

(d) When $0 \leq \beta \leq \alpha$, the plane cuts through both the nappes the curve of intersection is a hyperbola.

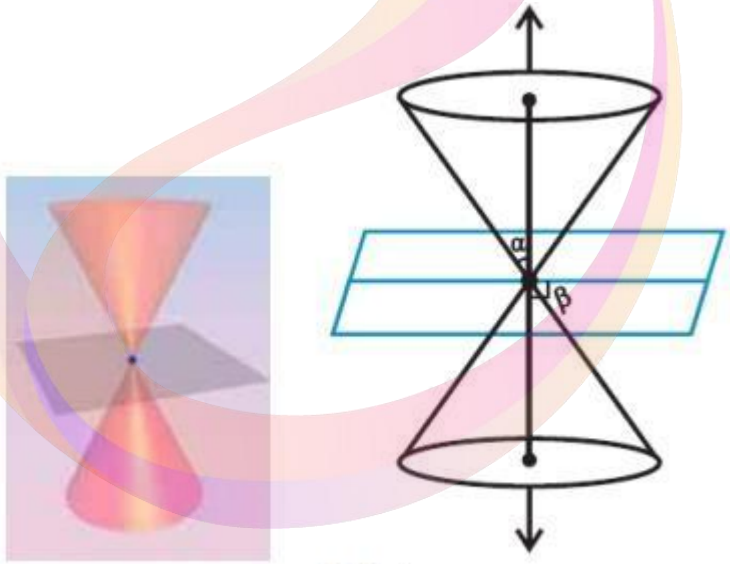



Hyperbola

The plane cuts both parts of the cone.

9. When the plane cuts at the vertex of the cone, we have the following different cases:

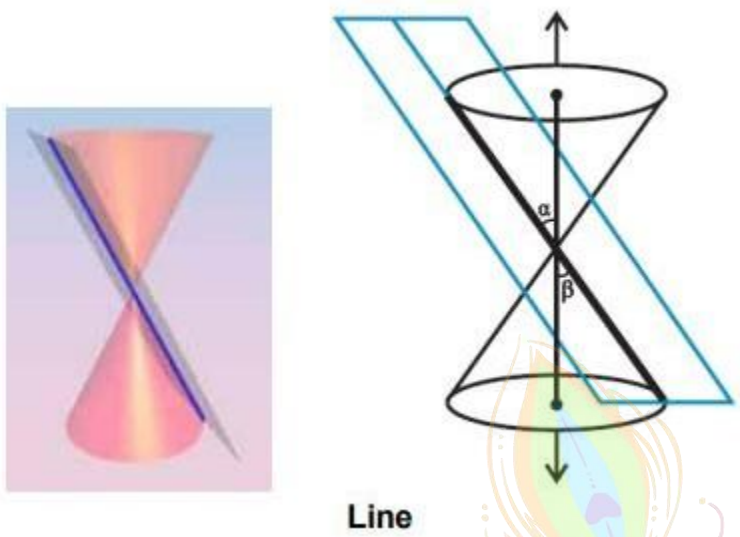
(a) When $\alpha < \beta < 90^{o}$, the section is a point.




Point

It is a degenerated case of a circle.

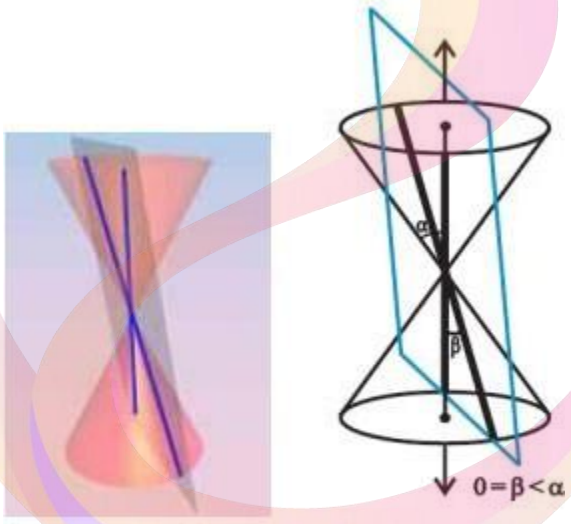
(b) When $\beta = \alpha$ the plane contains a generator of the cone and the section is a straight line.




Line

It is the degenerated case of parabola.

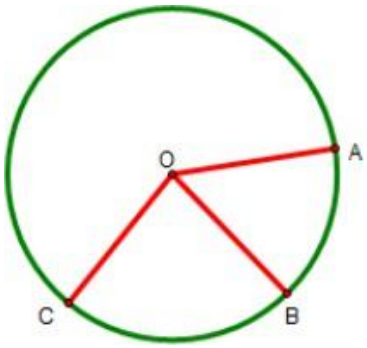
(c) When $0 \leq \beta \leq \alpha$, the section is a pair of intersecting straight lines. It is the degenerated case of ahyperbola.




Double Line

10. A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

11. The fixed point is called the center of the circle and the distance from the center to a point on thecircle is called the radius of the circle.




In the circle, O is the center and OA = OB = OC are the radii.

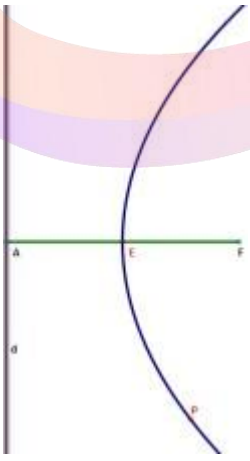
12. If the center of a circle is (h, k) and the radius is r, then the equation of the circle is given by $(x - h)^2 + (y - k)^2 = r^2$

13. A circle with the radius of length zero is a point circle.

14. If the center of a circle is at the origin and radius is r, then the equation of the circle is given by $x^2 + y^2 = r^2$.

15. If three points lie on the circle and if we prove that the fourth point also lies on the circle, then the fourpoints are concyclic.

16. A **parabola** is the locus of a point, which moves in a plane in such a way that its distance from a fixedpoint (not on the line) in the plane is equal to its distance from a fixed straight line in the same plane.



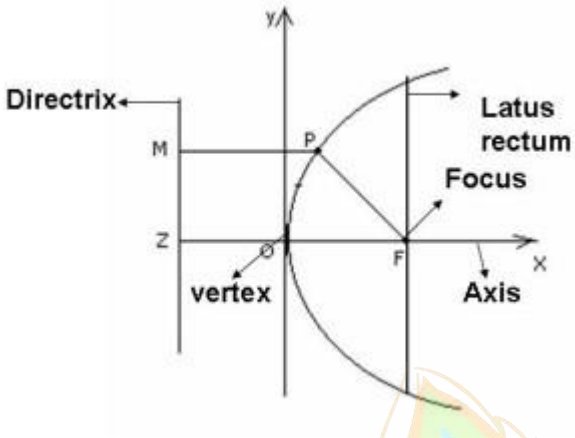
17. If the fixed point is on the fixed line, then the set of points which are equidistant from the line and focus will be a straight line which passes through the fixed point focus and perpendicular to the givenline. This straight line is the degenerate case of the parabola.


18. The fixed line is called the directrix of the parabola and the fixed point F is called the focus.

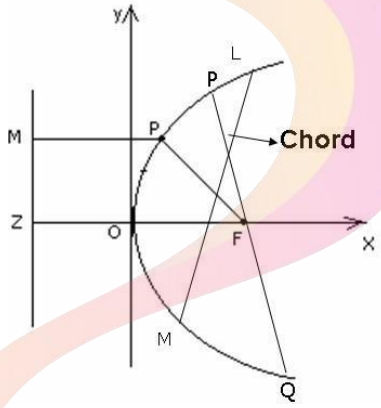
19. 'Para' means 'for' and 'bola' means 'throwing'. The path taken by the trajectory of a rocket artilleryetc. are parabolic.

    One of nature's best known approximations to parabolas is the path taken by a body projectedupward and obliquely to the pull of gravity, as in the parabolic trajectory of a golf ball.

20. A line through the focus and perpendicular to the directrix is called the axis of the parabola. The pointof intersection of the parabola with the axis is called the vertex of the parabola.



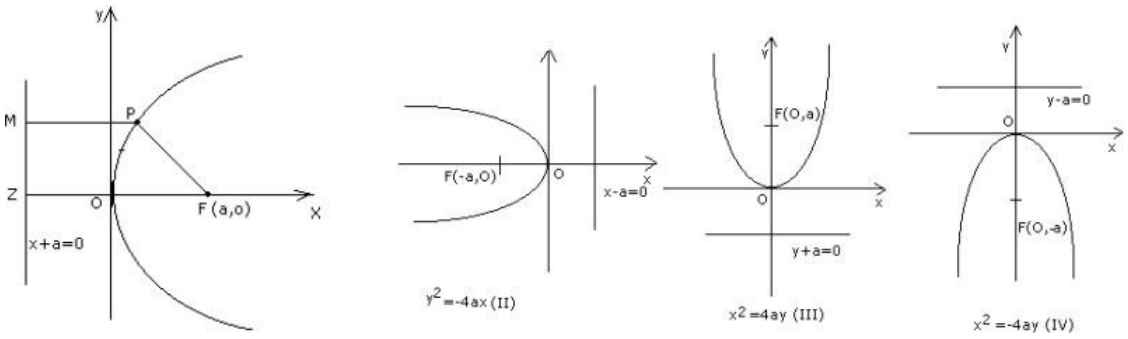

21. A chord of a parabola is the line segment joining any two points on the parabola. If the chord passesthrough the focus, then it is called the focal chord. LM and PQ are both chords but PQ is the focal chord.




22. The chord which passes through the focus is called the focal chord. Focal chord perpendicular to theaxis is called the **latus rectum** of the parabola.

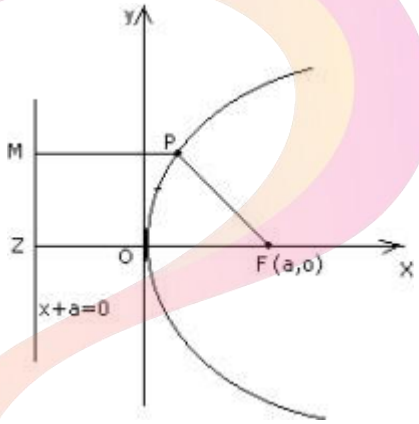
23. Any chord perpendicular to the axis of a parabola is called a double ordinate.

24. The equation of a parabola is simplest if the vertex is at the origin and the axis of symmetry is alongthe X-axis or Y-axis. The four such possible orientations of parabola are shown below.




25. In terms of loci, the conic sections can be defined as follows

    Given a line Z and point F not on Z, a conic is the locus of a point P such that the distance from P to Fdivided by the distance from P to Z is constant, i.e. PF/PM = e, which is a constant called eccentricity. In case of parabola eccentricity, e =1.




26. Parabola is symmetric with respect to its axis. If the equation has a $y^2$ term, then the axis of symmetryis along the X-axis and if the equation has an $x^2$ term, then the axis of symmetry is along the Y-axis.

27. When the axis of symmetry is along the X-axis, the parabola opens to the

    (a) Right if the coefficient of x is positive.

    (b) Left if the coefficient of x is negative.