NOTES

- NOTES WITH MIND MAPS -

MATHEMATICS

(COMPLEX NUMBERS AND QUADRATIC EQUATIONS)

11TH

COMPLEX NUMBERS & QUADRATIC EQUATIONS

Some Important Results

1. Solution of $x^2 + 1 = 0$ with the property $i^2 = -1$ is called the imaginary unit.
2. Square root of a negative real number is called an imaginary number.
3. If a and b are positive real numbers, then $\sqrt{-a} \times \sqrt{-b} = -\sqrt{ab}$
4. If a is a positive real number, then we have $\sqrt{-a} = i\sqrt{a}$.
5. Powers of i

   $i = \sqrt{-1};$

   $i^2 = -1;$

   $i^3 = -i$

   $i^4 = 1$
6. If n>4, then $i^{-n} = \frac{1}{i^n} = \frac{1}{i^k}$ where k is the reminder when n is divided by 4.
7. We have $i^0 = 1$.
8. A number in the form a + ib, where a and b are real numbers, is said to be a complex number.
9. In complex number z = a + ib, a is the real part, denoted by Re z and b is the imaginary part denoted by Im z of the complex number z.
10. $\sqrt{-1} = i$ is called iota, which is a complex number.
11. The modulus of a complex number z = a + ib denoted by |z| is defined to be a non- negative real number $\sqrt{a^2 + b^2}$, i.e. $|z| = \sqrt{a^2 + b^2}$.
12. For any non-zero complex number z = a + ib (a ≠ 0, b ≠ 0), there exists a complex number $\frac{a}{a^2+b^2} + i\frac{(-b)}{a^2+b^2}$, denoted by $\frac{1}{z}$ or z^{-1}, called the multiplicative inverse of z such that

$$(a+ib)\times\left(\frac{a}{a^2+b^2} + i\frac{(-b)}{a^2+b^2}\right) = 1 + i0 = 1.$$

13. Conjugate of a complex number z = a + ib, denoted as $\bar{z}$, is the complex number a − ib.
14. The number z = r(cos θ +isin θ) is the polar form of the complex number z = a + ib.

   Here $r = \sqrt{a^2+b^2}$ is called the modulus of z $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ and is called the argument or amplitude of z, which is denoted by arg z.

15. The value of θ such that $-\pi < \theta \leq \pi$ called principal argument of z.

16. The Eulerian form of z is $z = re^{i\theta}$, where $e^{i\theta} = \cos\theta + i\sin\theta$

17. The plane having a complex number assigned to each of its points is called the Complex plane orArgand plane.

18. Let $a_0, a_1, a_2, \ldots$ be real numbers and x is a real variable. Then, the real polynomial of a real variable with real coefficients is given as

    $f(x) = a_0 + a_1x + a_2x^2 + \ldots a_nx^n$

19. Let $a_0, a_1, a_2, \ldots$ be complex numbers and x is a complex variable. Then, the real polynomial of a complex variable with complex coefficients is given as

    $f(x) = a_0 + a_1x + a_2x^2 + \ldots a_nx^n$

20. A polynomial $f(x) = a_0 + a_1x + a_2x^2 + \ldots a_nx^n$ is a polynomial of degree n.

21. Polynomial of second degree is called a quadratic polynomial.

22. Polynomials of degree 3 and 4 are known as cubic and biquadratic polynomials.

23. If f(x) is a polynomial, then f(x) = 0 is called a polynomial equation.

24. If f(x) is a qua

25. dratic polynomial, then f(x) = 0 is called a quadratic equation.

26. The general form of a quadratic equation is $ax^2 + bx + c = 0, a \neq 0$.

27. The values of the variable satisfying a given equation are called its roots.

28. A quadratic equation cannot have more than two roots.

29. Fundamental Theorem of Algebra states that 'A polynomial equation of degree n has n roots.'

Top Concepts

1. **Addition of two complex numbers**: If $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers, then the sum
$z_1 + z_2 = (a + c) + i(b + d)$.
2. Sum of two complex numbers is also a complex number. This is known as the closure property.
3. The addition of complex numbers satisfy the following properties:
   i. Addition of complex numbers satisfies the commutative law. For any two complex numbers z_1 and z_2, $z_1 + z_2 = z_2 + z_1$.
   ii. Addition of complex numbers satisfies associative law for any three complex numbers z_1, z_2, z_3, $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.
   iii. There exists a complex number 0 + i0 or 0, called the additive identity or the zero complex number, such that for every complex number z,
   $z + 0 = 0 + z = z$.
   iv. To every complex number $z = a + ib$, there exists another complex number $-z = -a + i(-b)$ called the additive inverse of z.
   $z+(-z)=(-z)+z=0$
4. **Difference of two complex numbers:** Given any two complex numbers If $z_1 = a + ib$ and $z_2 = c + id$ the difference $z_1 - z_2$ is given by
$z_1 - z_2 = z_1 + (-z_2) = (a - c) + i(b - d)$.
5. **Multiplication of two complex numbers** Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the product $z_1 z_2$ is defined as follows:
$z_1 z_2 = (ac - bd) + i(ad + bc)$
6. **Properties of multiplication of complex numbers**: Product of two complex numbers is a complex number; the product $z_1 z_2$ is a complex number for all complex numbers z_1 and z_2.
   i. Product of complex numbers is commutative, i.e. for any two complex numbers z_1 and z_2, $z_1 z_2 = z_2 z_1$
   ii. Product of complex numbers is associative, i.e. for any three complex numbers z_1, z_2, z_3, $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.

iii. There exists a complex number 1 + i0 (denoted as 1), called the multiplicative identitysuch that z.1 = z for every complex number z.

iv. For every non-zero complex number, z = a + ib or a + bi ($a \neq 0$, $b \neq 0$), there is a complex number $\frac{a}{a^2+b^2} + i\frac{-b}{a^2 \quad b^2}$ called the multiplicative inverse of z such that $z \times \frac{1}{z} = 1$

v. distributive law: For any three complex numbers z_1, z_2, z_3,

a. $z_1(z_2 + z_3) = z_1.z_2 + z_1.z_3$

b. $(z_1 + z_2)\, z_3 = z_1.z_3 + z_2.z_3$

7. **Division of two complex numbers**: Given any two complex numbers $z_1 = a + ib$ and $z_2 = c + id$, where $z_2 \neq 0$, the quotient $\frac{z_1}{z_2}$ is defined by

$$\frac{z_1}{z_2} = z_1.\frac{1}{} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2 \quad d^2}.$$

8. Identities for complex numbers

i. $(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1.z_2$, for all complex numbers z_1 and z_2.

ii. $(z_1 - z_2)^2 = z_1^2 - 2z_1z_2 + z_2^2$

iii. $(z_1 + z_2)^3 = z_1^3 + 3z_1^2z_2 + 3z_1z_2^2 + z_2^3$

iv. $(z_1 - z_2)^3 = z_1^3 - 3z_1^2z_2 + 3z_1z_2^2 - z_2^3$

v. $z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)$

9. Properties of modulus and conjugate of complex numbers

For any two complex numbers z_1 and z_2,

i. $|z_1 z_2| = |z_1||z_2|$

ii. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z|}$ provided $|z_2| \neq 0$

iii. $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

iv. $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

v. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ provided $z_2 \neq 0$

vi. $\overline{(\overline{z})} = z$

vii. $z + \overline{z} = 2\text{Re}(z)$

viii. $z - \overline{z} = 2i\,\text{Im}(z)$

ix. $z = \overline{z} \Leftrightarrow z$ is purely real

x. $z + \overline{z} = 0 \Rightarrow z$ is purely imaginary

xi. $z\overline{z} = [\text{Re}(z)]^2 + [\text{Im}(z)]^2$

10. The order of a relation is not defined in complex numbers. Hence there is no meaning in $z_1 >$ z.
11. Two complex numbers z_1 and z_2 are equal iff Re (z_1) = Re (z_2) and Im (z_1) Im (z_2).
12. The sum and product of two complex numbers are real if and only if they are conjugate of each other.
13. For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$. $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ when a<0 and b<0.
14. The polar form of the complex number z = x + iy is r (cos θ + i sinθ), where r is the modulus of z and θ is known as the argument of z.
15. For a quadratic equation $ax^2 + bx + c = 0$ with real coefficients a, b and c and $a \neq 0$. If the discriminant $D = b^2 - 4ac \geq 0$, then the equation has two real roots given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b}{2a}.$$

16. Roots of the quadratic equation $ax^2 + bx + c = 0$, where a, b and c $\in$ R, $a \neq 0$, when discriminant $b^2 - 4ac < 0$, are imaginary given by

$$x = \frac{-b \pm \sqrt{4ac - b^2}i}{2a}.$$

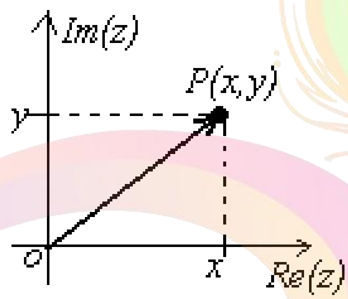
17. Complex roots occur in pairs.
18. If a, b and c are rational numbers and $b^2 - 4ac$ is positive and a perfect square, then $\sqrt{b^2 - 4ac}$

is a rational number and hence the roots are rational and unequal.

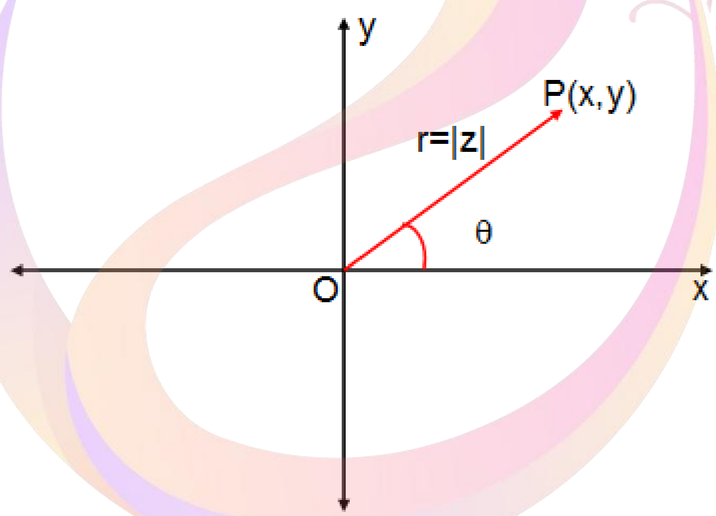
19. If b^2 - 4ac = 0, then the roots of the quadratic equation are real and equal.

20. If b^2 - 4ac = 0 but it is not a perfect square, then the roots of the quadratic equation are irrational and unequal.

21. Irrational roots occur in pairs.
22. A polynomial equation of n degree has n roots. These n roots could be real or complex.
23. Complex numbers are represented in the Argand plane with X-axis being real and Y-axis being imaginary.

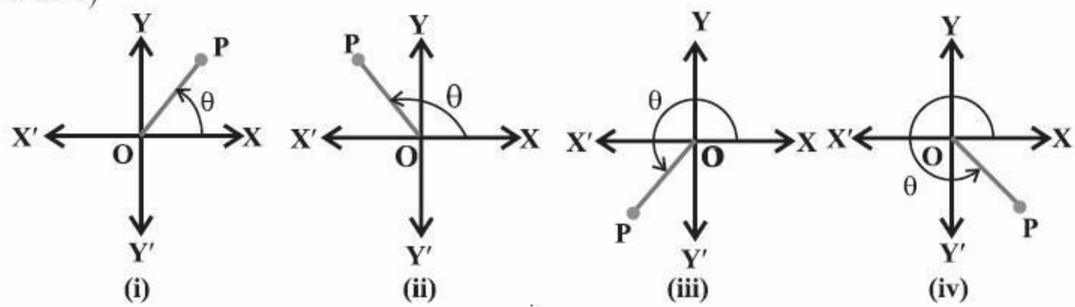


24. Representation of complex number z = x + iy in the Argand plane.



25. Multiplication of a complex number by i results in rotating the vector joining the origin to the pointrepresenting z through a right angle.
26. Argument θ of the complex number z can take any value in the interval [0, 2π). Different orientations of z are as follows

$(0 \le \theta < 2\pi)$



$(-\pi < \theta \le \pi)$