NOTES

- NOTES WITH MIND MAPS -

MATHEMATICS

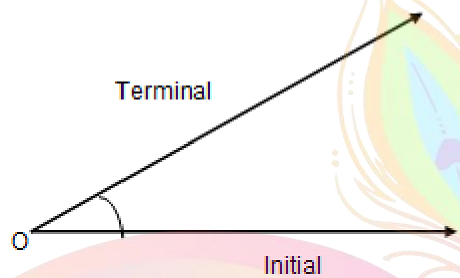
(TRIGONOMETRIC FUNCTIONS)

11TH

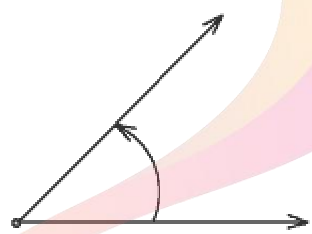
TRIGONOMETRIC FUNCTION

Top Concepts

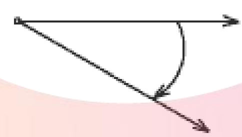
1. An angle is a measure of rotation of a given ray about its initial point. The original position of the ray before rotation is called the initial side of the angle, and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex.



2. If the direction of rotation is anticlockwise, then the angle is said to be positive, and if the direction ofrotation is clockwise, then the angle is negative.

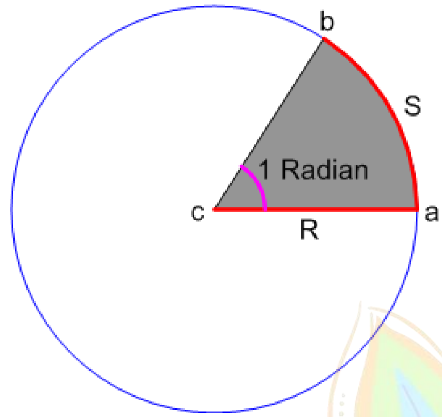


Positive angle—Anticlockwise



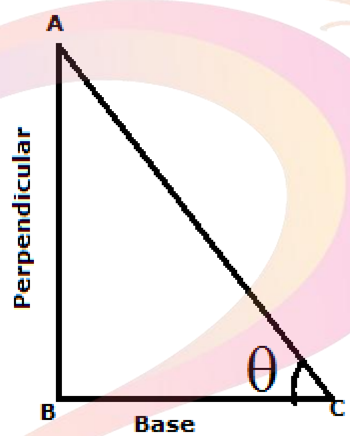
Negative angle—Clockwise

3. If a rotation from the initial side to terminal side is $(\frac{1}{360})^{th}$ of a revolution, then the angle is said to have a measure of one degree. It is denoted by 1°.
4. A degree is divided into 60 minutes, and a minute is divided into 60 seconds. One sixtieth of a degreeis called a minute and is written as 1′, and one sixtieth of a minute is called a Second and is written as1″.
   Thus, 1° = 60′ and 1′ = 60″.
5. The angle subtended at the centre by an arc of length 1 unit in a unit circle is said to have a measure of 1 radian.



6. Basic trigonometric ratios

Consider the following triangle:



i. Sineθ = $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$

ii. Cosineθ = $\frac{\text{Base}}{\text{Hypotenuse}}$

iii. Tangentθ = $\frac{\text{Perpendicular}}{\text{Base}}$

iv. Cosecantθ = $\frac{\text{Hypotenuse}}{\text{Perpendicular}}$

v. Secantθ = $\frac{\text{Hypotenuse}}{\text{Base}}$

vi. Cotangentθ = $\frac{\text{Base}}{\text{Perpendicular}}$

7. Some trigonometric identities

i. $\sin\theta = \frac{1}{\text{cosec}}$ or $\text{cosec}\theta = \frac{1}{\sin\theta}$

ii. $\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

iii. $\tan\theta = \frac{1}{\cot}$ or $\cot\theta = \frac{1}{\tan\theta}$

iv. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

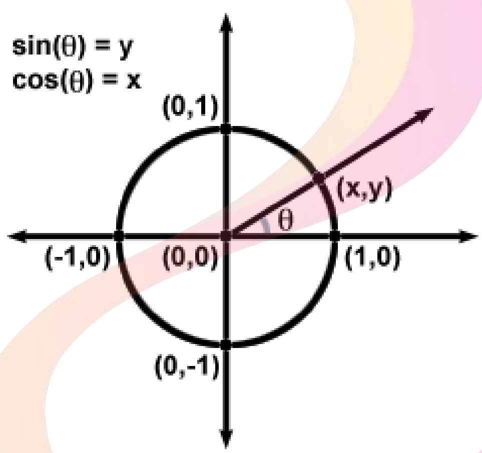
v. $\cot\theta = \frac{\cos\theta}{\sin\theta}$

vi. $\sin^2\theta + \cos^2\theta = 1$

vii. $1 + \tan^2\theta = \sec\ \theta$

viii. $1 + \cot^2\theta = \text{cosec}\ \theta$

8. If a point on the unit circle is on the terminal side of an angle in the standard position, then the sine of such an angle is simply the y-coordinate of the point and the cosine of the angle is the x-coordinate of that point.



9. All the angles which are integral multiples of $\frac{\pi}{2}$ are called quadrantal angles. Values of quadrantal angles are:

   $\cos 0 = 1, \sin 0 = 0$

   $\cos \frac{\pi}{2} = 0, \sin \frac{\pi}{2} = 1$

   $\cos \pi = -1, \sin \pi = 0$

   $\cos \frac{3\pi}{2} = 0, \sin \frac{3\pi}{2} = -1$

   $\cos 2\pi = 1, \sin 2\pi = 0$

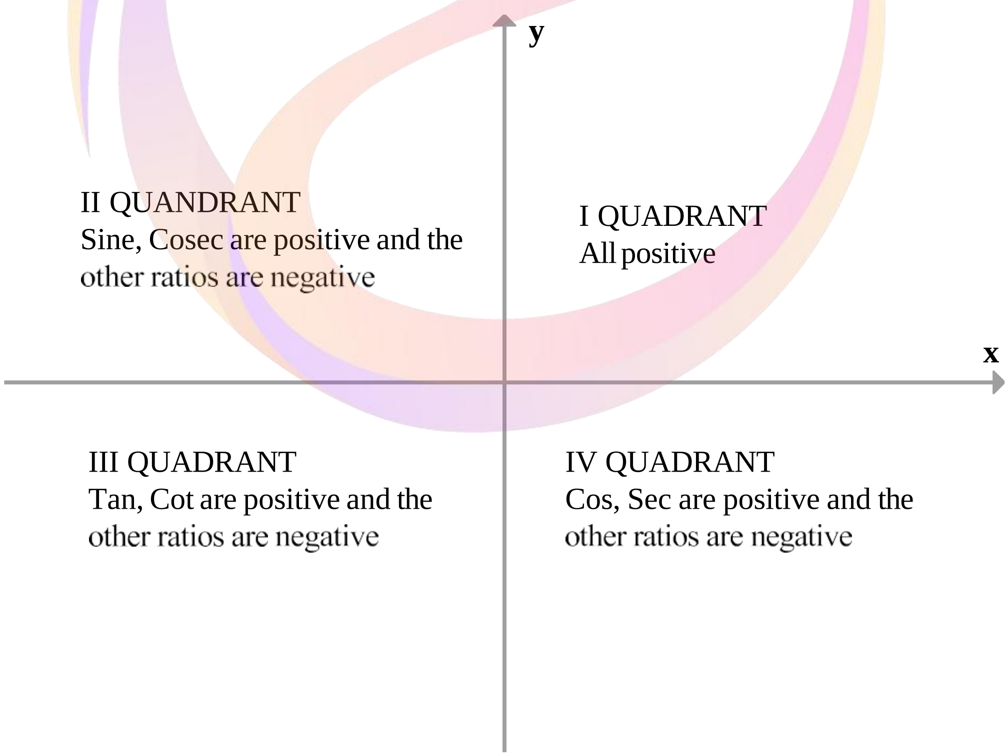
10. Even function: A function f(x) is said to be an even function if f(-x) = f(x) for all x in its domain.
11. Odd function: A function f(x) is said to be an odd function if f(-x) = -f(x) for all x in its domain.
12. Cosine is even and sine is an odd function

    $\cos(-x) = \cos x$

    $\sin(-x) = -\sin x$
13. Signs of trigonometric functions in various quadrants

    In quadrant I, all the trigonometric functions are positive.

    In quadrant II, only sine is positive. In quadrant III, only tan is positive. In quadrant IV, only cosine function is positive. This is depicted as follows



14. In quadrants where Y-axis is positive (i.e. I and II), sine is positive, and in quadrants where X-axis ispositive (i.e. I and IV), cosine is positive.
15. A simple rule to remember the signs of trigonometrical ratios in all the four quadrants is the four letter phrase—**A**ll **S**chool **T**o **C**ollege.
16. A function f is said to be a periodic function if there exists a real number T>0 such that f(x + T) = f(x) for all x. This T is the period of function.
17. Trigonometric ratios of complementary angles
    i. $\sin(90^o - \theta) = \cos\theta$
    ii. $\cos(90^o - \theta) = \sin\theta$
    iii. ten$(90^o - \theta) = \cot\theta$
    iv. $\text{cosec}(90^o - \theta) = \sec\theta$
    v. $\sec(90^o - \theta) = \text{cosec}\theta$
    vi. $\cot(90^o - \theta) = \tan\theta$
18. Trigonometric ratios of $(90^o + \theta)$ in terms of θ
    i. $\sin(90^o + \theta) = \cos\theta$
    ii. $\cos(90^o + \theta) = -\sin\theta$
    iii. ten$(90^o + \theta) = -\cot\theta$
    iv. $\text{cosec}(90^o + \theta) = \sin\theta$
    v. $\sec(90^o + \theta) = \text{cosec}\theta$
    vi. $\cot(90^o + \theta) = -\tan\theta$
19. Trigonometric ratios of $(180^o - \theta)$ in terms of θ
    i. $\sin(180^o - \theta) = \sin\theta$
    ii. $\cos(180^o - \theta) = -\cos\theta$
    iii. ten$(180^o - \theta) = -\tan\theta$
    iv. $\text{cosec}(180^o - \theta) = \text{cosec}\theta$
    v. $\sec(180^o - \theta) = -\sec\theta$
    vi. $\cot(180^o - \theta) = -\cot\theta$
20. Trigonometric ratios of $(180^o + \theta)$ in terms of θ
    i. $\sin(180^o + \theta) = -\sin\theta$
    ii. $\cos(180^o + \theta) = -\cos\theta$
    iii. ten$(180^o + \theta) = \tan\theta$
    iv. $\text{cosec}(180^o + \theta) = -\text{cosec}\theta$
    v. $\sec(180^o + \theta) = -\sec\theta$
    vi. $\cot(180^o + \theta) = \cot\theta$
21. Trigonometric ratios of $(360^o - \theta)$ in terms of θ
    i. $\sin(360^o - \theta) = -\sin\theta$
    ii. $\cos(360^o - \theta) = \cos\theta$

iii. ten(360° - θ) = -tanθ
iv. cosec(360° - θ) = -cosecθ
v. sec(360° - θ) = secθ
vi. cot(360° - θ) = -cotθ

22. Trigonometric ratios of (360° + θ) in terms of θ

i. sin(360° + θ) = sinθ
ii. cos(360° + θ) = cosθ
iii. ten(360° + θ) = tanθ
iv. cosec(360° + θ) =-cosecθ
v. sec(360° + θ) = secθ
vi. cot(360° + θ) = cotθ

23. In the case of sin (2π + x) = sin x, so the period of sine is 2π. Period of its reciprocal is also 2π.
24. In the case of cos (2π + x) = cos x, so the period of cos is 2π. Period of its reciprocal is also 2π.
25. In the case of tan (π + x) = tan x. Period of tangent and cotangent function is π.
26. The graph of cos x can be obtained by shifting the sine function by the factor $\frac{\pi}{2}$.
27. The tan function differs from the previous two functions in two ways

(i) Function tan is not defined at the odd multiples of $\frac{\pi}{2}$.

(ii) The tan function is not bounded.

28.

Function	**Period**
y = sin x	2π
y = sin (ax)	$\frac{2\pi}{a}$
y = cos x	2π
y = cos (ax)	$\frac{2\pi}{a}$
y = cos 3x	$\frac{2\pi}{3}$
y = sin 5x	$\frac{2\pi}{5}$

29. For a function of the form

y= kf(ax + b)

the range will be k times the range of function x, where k is any real number.

If f(x) = sine or cosine function, the range will be equal to R−[−k, k].
If the function is of the form sec x or cosec x, the period is equal to the period of function f by a.
The position of the graph is b units to the right/left of y = f(x) depending on whether b>0 or b<0.

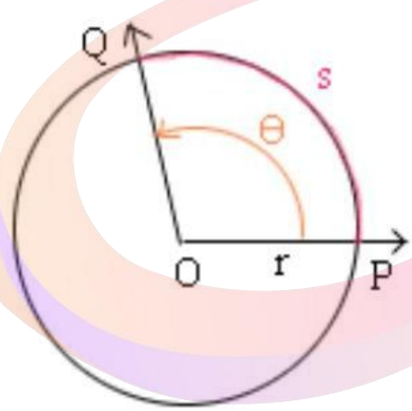
30. The solutions of a trigonometric equation, for which $0 \le x \le 2\pi$, are called principal solutions.
29. The expression involving integer 'n' which gives all solutions of a trigonometric equation is called thegeneral solution.
30. The numerical smallest value of the angle (in degree or radian) satisfying a given trigonometric equation is called the Principal Value. If there are two values, one positive and the other negative,which are numerically equal, then the positive value is taken as the Principal Value.

Top Formulae

1. 1 radian = $\frac{180^0}{\pi}$ = 57°16' approximately.

2. 1° = $\frac{180^0}{\pi}$ radians = 0.01746 radians approximately.

3.



s= r θ

Length of arc = radius × angle in radian.

This relation can only be used when θ is in radians.

3. Radian measure = $\frac{\pi}{180}$ × degree measure.

4. degree measure = $\frac{180}{\pi}$ × Radian measure.

5. Values of trigonometric ratios:

	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0
cosec	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not defined	–1	Not defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	–1	Not defined	1
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0	Not defined

6. Domain and range of various trigonometric functions:

Function	**Domain**	**Range**
y = sin x	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	[–1, 1]
y = cos x	$[0, \pi]$	[–1, 1]
y = cosec x	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	R – (–1,1)
y = sec x	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	R – (–1, 1)
y = tan x	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	R
y = cot x	$(0, \pi)$	R