NOTES

– NOTES WITH MIND MAPS –

MATHEMATICS

(RELATIONS AND FUNCTIONS)

RELATIONS AND FUNCTIONS

Key Concepts

1. A pair of elements grouped together in a particular order is known as an ordered pair.

2. The two ordered pairs (a, b) and (c, d) are said to be equal if and only if a = c and b = d.

3. Let A and B be any two non-empty sets. The Cartesian product A × B is the set of all ordered pairs ofelements of sets from A and B defined as follows:
   $A \times B = \{(a, b) : a \in A, b \in B\}$.
   Cartesian product of two sets is also known as the product set.

4. If any of the sets of A or B or both are empty, then the set A × B will also be empty and consequently, n(A × B) = 0.

5. If the number of elements in A is m and the number of elements in set B is n, then the set A × B willhave mn elements.

6. If any of the sets A or B is infinite, then A × B is also an infinite set.

7. Cartesian product of sets can be extended to three or more sets. If A, B and C are three non-emptysets, then $A \times B \times C = \{(a, b, c): a \in A, b \in B, c \in C\}$. Here (a, b, c) is known as an ordered triplet.

8. Cartesian product of a non-empty set A with an empty set is an empty set, i.e. $A \times \Phi = \Phi$.

9. The Cartesian product is not commutative, namely A × B is not the same as B × A, unless A and B are equal.

10. The Cartesian product is associative, namely $A \times (B \times C)=(A \times B) \times C$

11. $R \times R = \{(a, b) : a \in R, b \in R\}$ represents the coordinates of all points in two-dimensional plane. $R \times R \times R = \{(a, b, c): a \in R, b \in R, c \in C\}$ represents the coordinates of all points in three-dimensional plane.

12. A relation R from the non-empty set A to another non-empty set B is a subset of their Cartesianproduct A × B, i.e. $R \subseteq A \times B$.

13. If $(x, y) \in R$ or x R y, then x is related to y.

14. If $(x, y) \notin R$ or $x \not R y$, then x is not related to y.

15. The second element b in the ordered pair (a, b) is the image of first element a and a is the pre-image of b.

16. The **Domain** of R is the set of all first elements of the ordered pairs in a relation R. In other words,domain is the set of all the inputs of the relation.

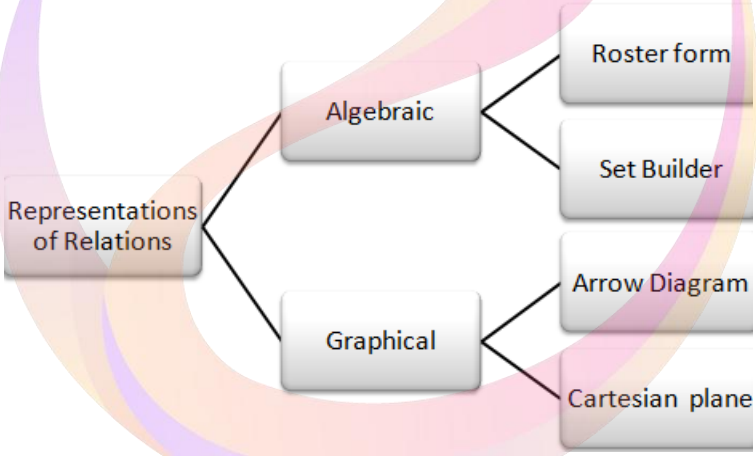
17. If the relation R is from a non-empty set A to non-empty set B, then set B is called the **co-domain** ofrelation R.

18. The set of all the images or the second element in the ordered pair (a, b) of relation R is called the **Range** of R.

19. The total number of relations that can be defined from a set A to a set B is the number is possiblesubsets of A × B.

20. A × B can have 2^{mn} subsets. This means there are 2^{mn} relations from A to B

21. Relation can be represented algebraically and graphically. The various methods are as follows



22. A relation f from a non-empty set A to another non-empty set B is said to be a function if everyelement of A has a unique image in B.

23. The domain of f is the set A. No two distinct ordered pairs in f have the same first element.

24. Every function is a relation but the converse is not true.

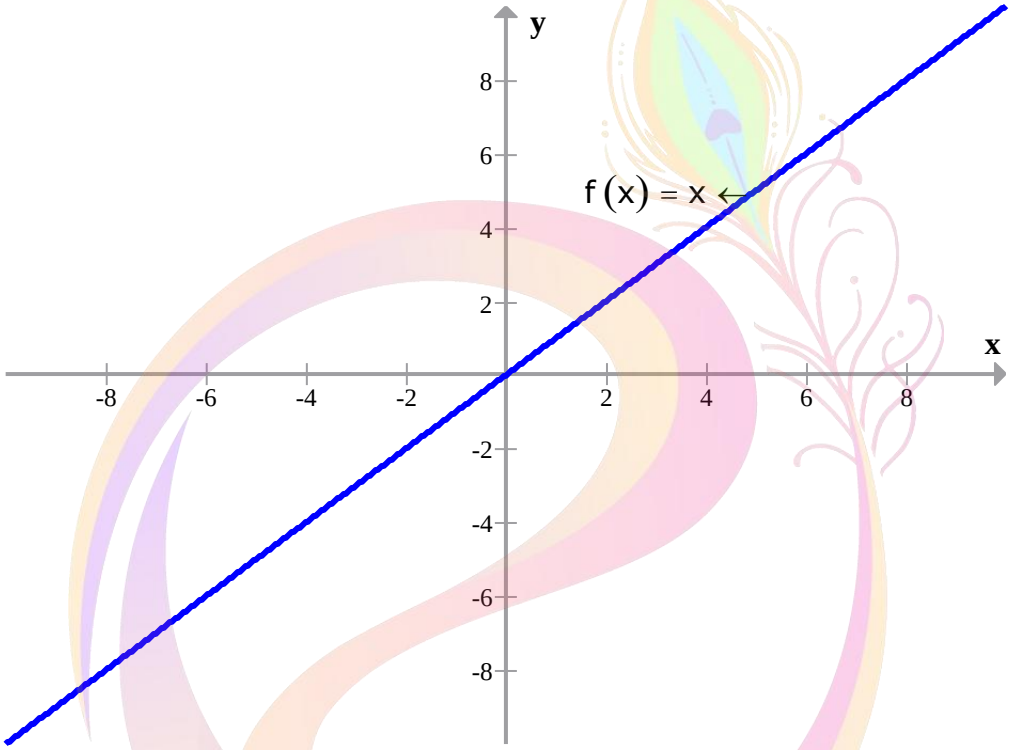
25. If f is a function from A to B and (a, b) ∈ f, then f (a) = b, where b is called **image** of a under f and a iscalled the **pre-image** of b under f.

26. If f: A $\rightarrow$ B A is the domain and B is the co domain of f.

27. The range of the function is the set of images.

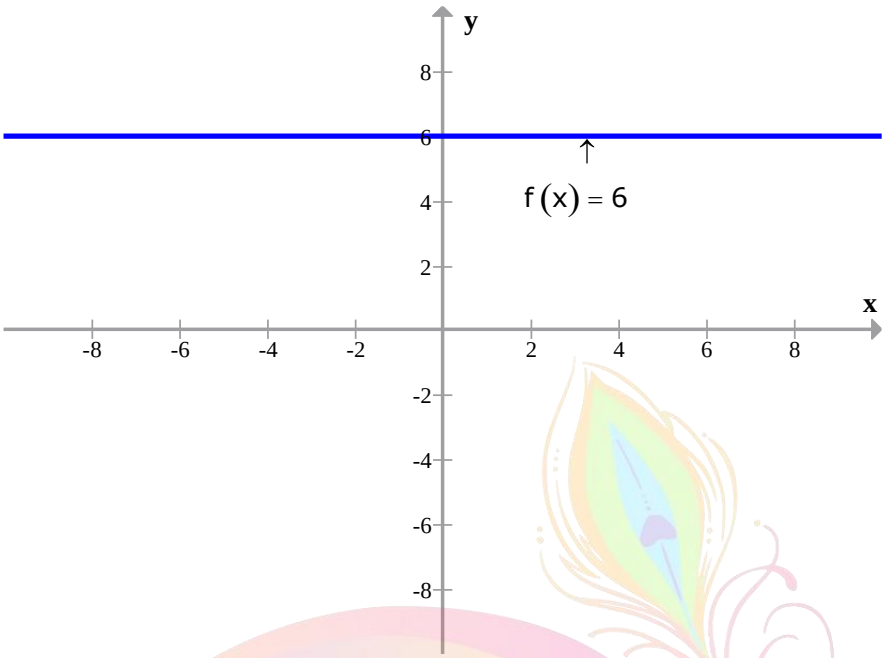
28. A real function has the set of real numbers or one of its subsets both as its domain and as its range.

29. **Identity function**: f: X $\rightarrow$ X is an identity function if f(x) = x for each x $\in$ A



30. Graph of the identity function is a straight line that makes an angle of 45° with both X- and Y–axis,respectively. All points on this line have their x and y coordinates equal.

31. **Constant function**: A constant function is one that maps each element of the domain to a constant. Domain of this function is R and range is the singleton set {c}, where c is a constant.



32. Graph of the constant function is a line parallel to the X-axis. The graph lies above X-axis if theconstant $c > 0$, below the X-axis if the constant $c < 0$ and is the same as X-axis if $c = 0$.

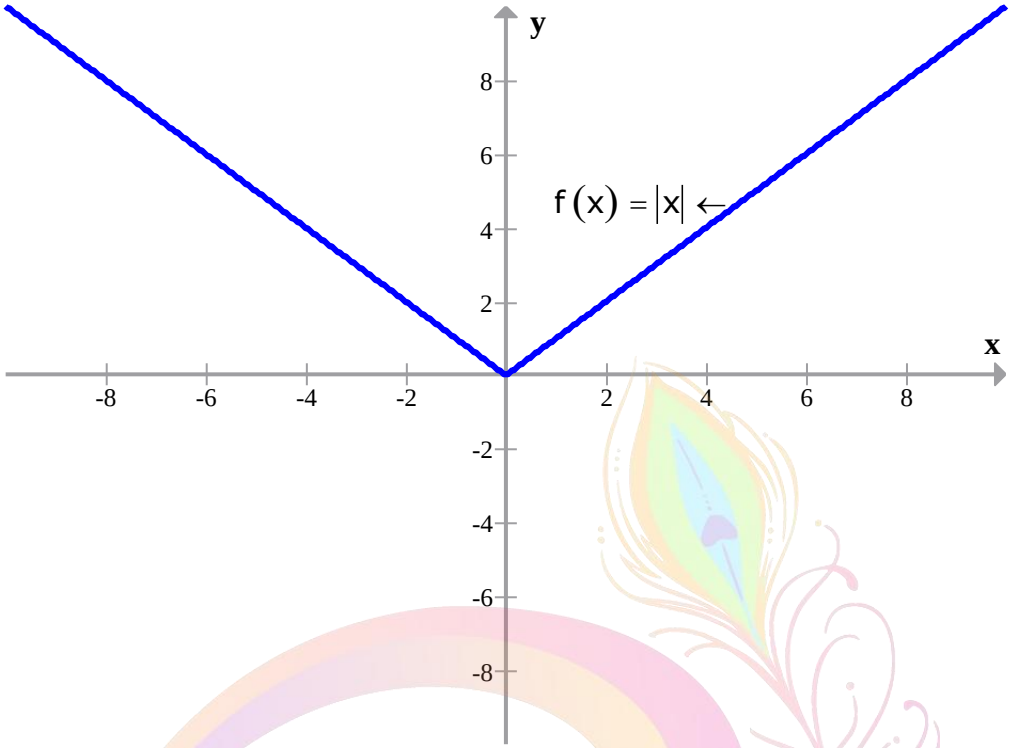
33. **Polynomial function**: f: R $\rightarrow$ R defined as $y = f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_n x^n$, where n is a non-negative integer and $a_0, a_1, a_2, \ldots a_n \in R$.

34. A linear polynomial represents a straight line, while a quadratic polynomial represents a parabola.

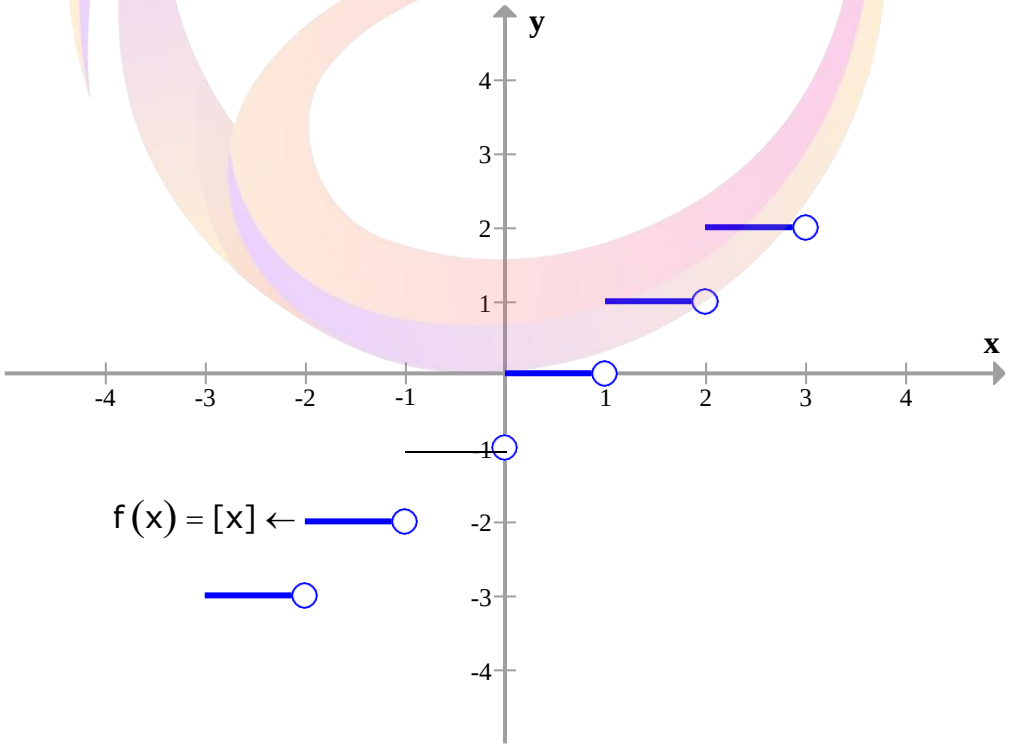
35. Functions of the form $\frac{f(x)}{g(x)}$, where f(x) and $g(x) \neq 0$ are polynomial functions, are called rational functions.

36. Domain of rational functions does not include those points where g(x) = 0. For example, the domain of $f(x) = \frac{1}{x-2}$ is $R - \{2\}$.

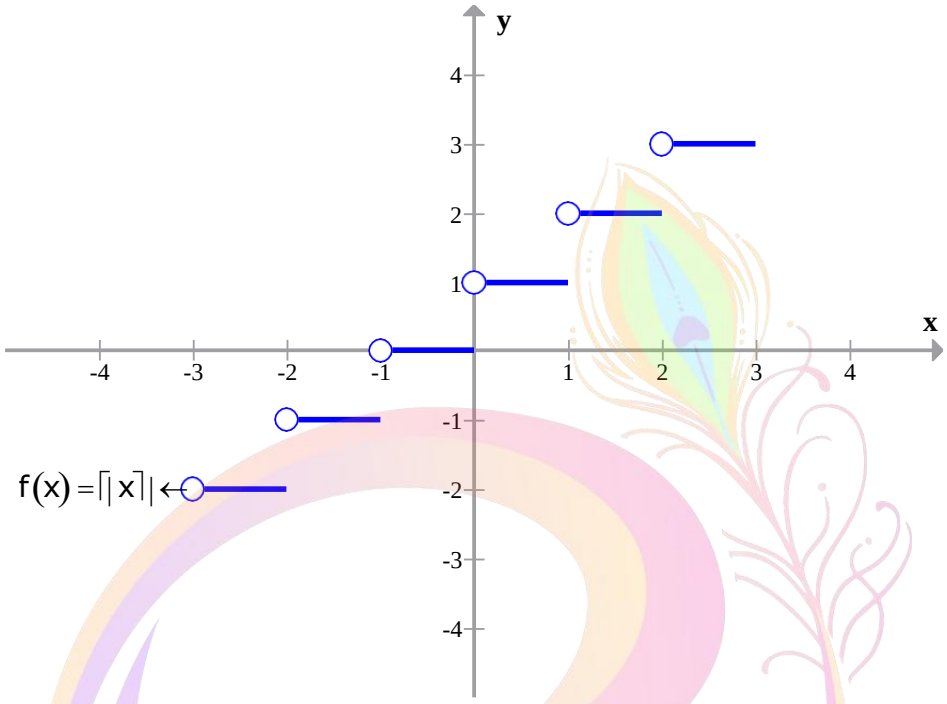
37. **Modulus function**: f: R $\rightarrow$ R defined by $f(x) = |x|$ for each $x \in R$
$f(x) = x$ if $x \geq 0$ $f(x) = -x$ if $x<0$ is called the modulus or absolute value function. The graph of modulusfunction is above the X-axis.



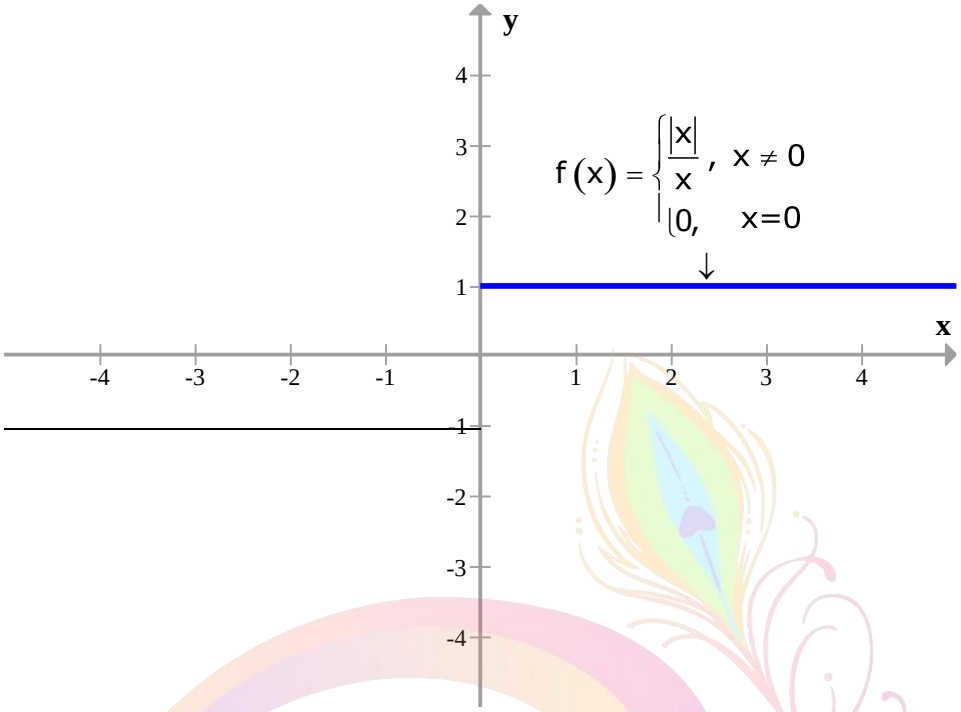
38. **Step or greatest integer function**: A function f: R $\rightarrow$ R defined by f(x) = [x], x $\in$ R, where [x] is the value of greatest integer, less than or equal to x is called a step or greatest integer function. It is alsocalled as floor function.



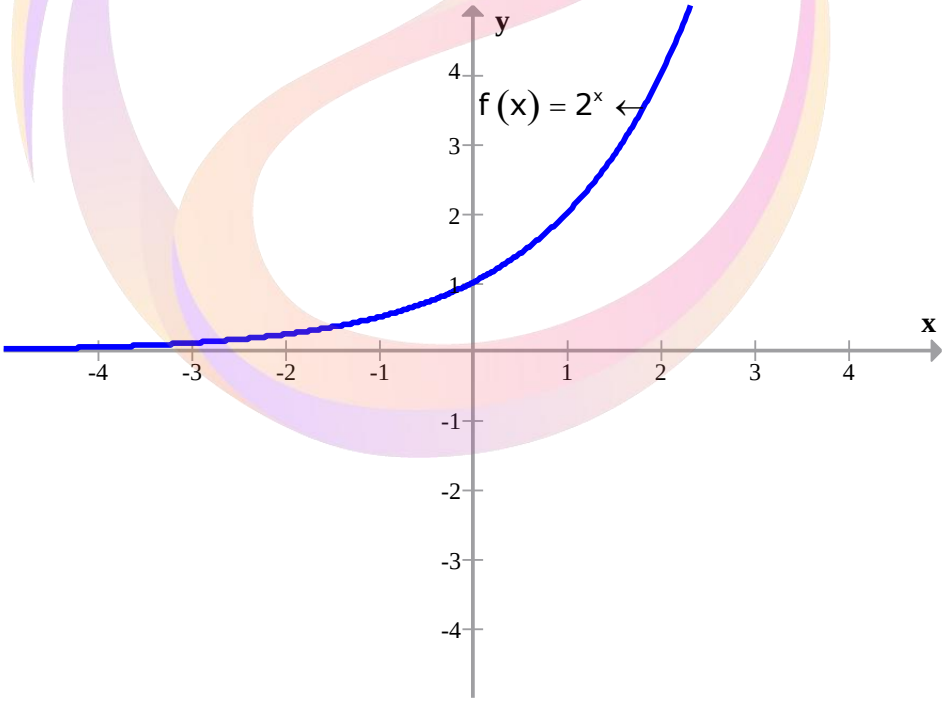
39. Smallest integer function: A function f: R $\rightarrow$ R defined by f(x) = [x], x$\in$ R where smallest integer, greater than or equal to x is called a smallest integer function. It is also known as theceiling function.



40. Signum function: $f(x) = \frac{|x|}{x}$, $x \neq 0$ and 0 for x = 0. The domain of signum function is R and range is {−1, 0, 1}.

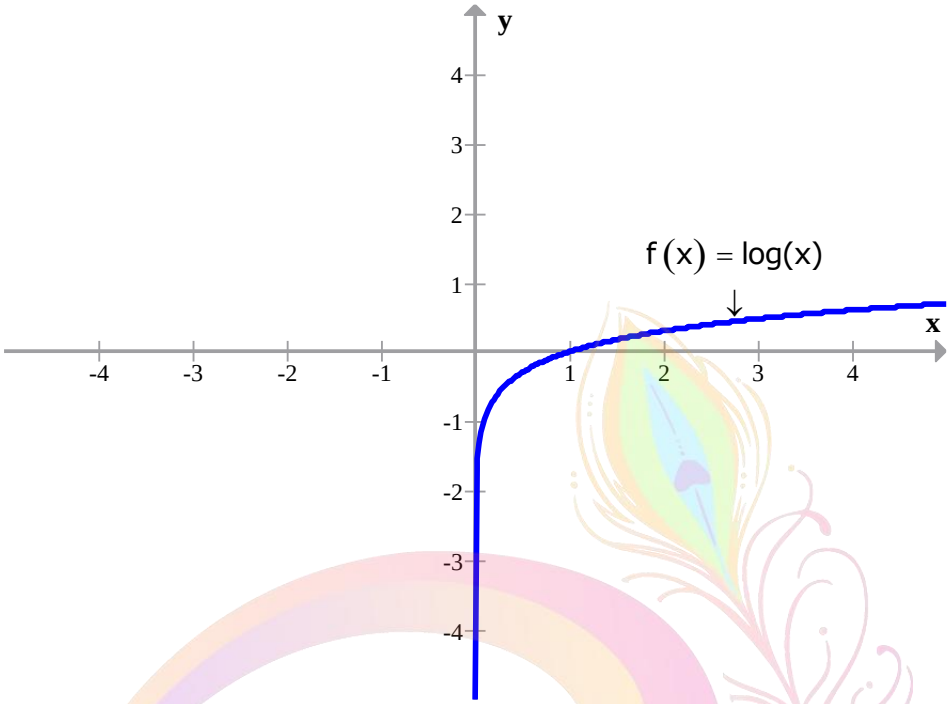


41. If a is a positive real number other than unity, then a function that relates each $x \in R$ to a^x is calledthe exponential function.

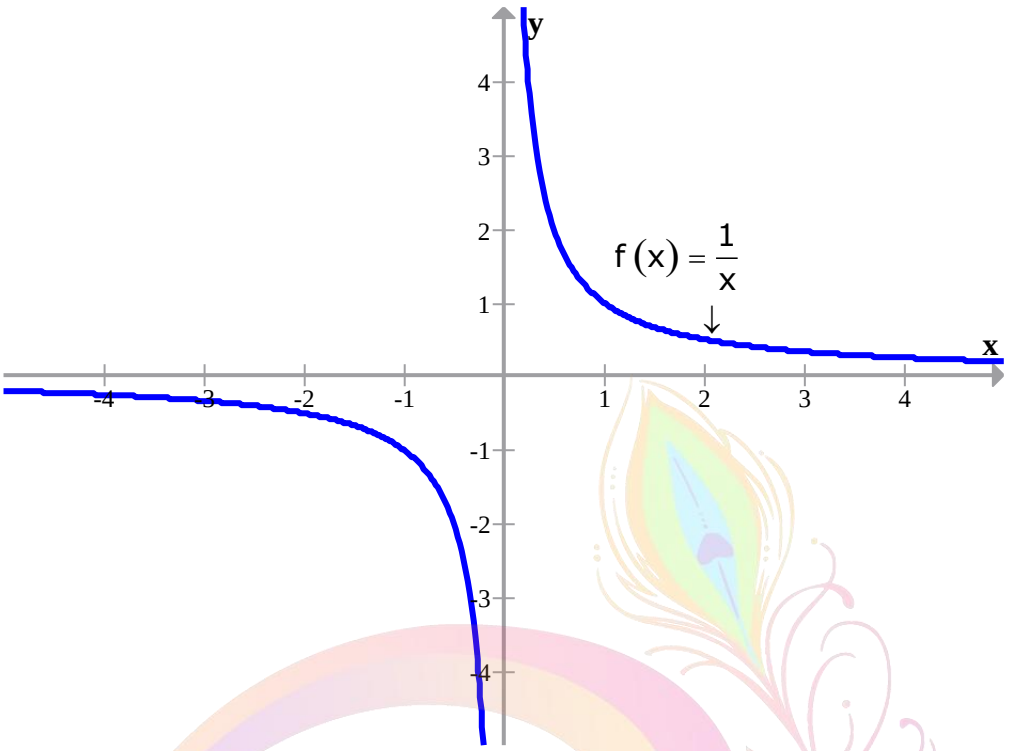


42. If $a > 0$ and $a \neq 1$, then the function defined by $f(x) = \log_a x, x > 0$ is called the

logarithmicfunction.



43. The function defined by f : R – {0} → R such that, f(x)= $\frac{1}{x}$ is called the reciprocal function



44. The function defined by $f:R^{+} \to R$ such that, $f(x) = +\sqrt{x}$ is called the square root function.