NOTES

- NOTES WITH MIND MAPS -

MATHEMATICS

(INTRODUCTION TO TRIGONOMETRY)

10TH

Introduction to Trigonometry

1. Meaning (Definition) of Trigonometry

The word trigonometry is derived from the Greek words ‘*tri*’ meaning three, ‘*gon*’ meaning sides and ‘metron’ meaning measure.

Trigonometry is the study of relationships between the sides and the angles of the triangle.

2. Positive and negative angles

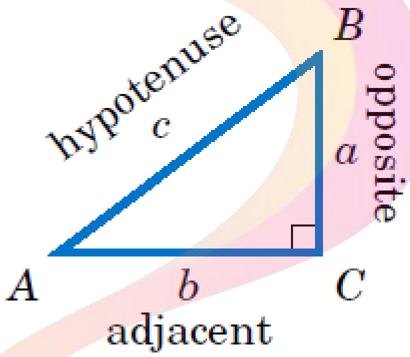
Angle measured in anticlockwise direction is taken as positive angle whereas the angle measured in clockwise direction is taken as negative angle.

3. Trigonometric Ratios

Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios**

of the angle.

Trigonometric ratios of the acute angle A in right triangle ABC are given as follows:



i. $\sin\angle A = \dfrac{\text{side opposite to}\angle A}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{p}{h}$

ii. $\cos\angle A = \dfrac{\text{side adjacent to}\angle A}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{b}{h}$

iii. $\tan\angle A = \dfrac{\text{side opposite to}\angle A}{\text{side adjacent to}\angle A} = \dfrac{BC}{AB} = \dfrac{p}{b}$

iv. $\text{cosec}\angle A = \dfrac{\text{hypotenuse}}{\text{side opposite to}\angle A} = \dfrac{AC}{BC} = \dfrac{h}{p}$

v. $\sec\angle A = \dfrac{\text{hypotenuse}}{\text{side adjacent to}\angle A} = \dfrac{AC}{AB} = \dfrac{h}{b}$

vi. $\cot\angle A = \dfrac{\text{side adjacent to}\angle A}{\text{side opposite to}\angle A} = \dfrac{AB}{BC} = \dfrac{b}{p}$

4. Important facts about Trigonometric ratios

- Trigonometric ratios of an acute angle in a right triangle represents the relation between the angle and the sides.
- The ratios defined above can be rewritten as sin A, cos A, tan A, cosec A, sec A and cot A.

- Each trigonometric ratio is a real number and it has not unit.
- All the trigonometric symbols i.e., cosine, sine, tangent, cotangent, secant and cosecant, have no literal meaning.
- $(\sin\theta)^n$ is generally written as $\sin^n \theta$, *n* being *a* positive integer. Similarly, other trigonometric ratios can also be written.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

5. **Pythagoras theorem:**

It states that "in a right triangle, square of the hypotenuse is equal to the sum of the squar es of the other two sides".

Pythagoras theorem can be used to obtain the length of the side of a right angled triangle when the other two sides are already given.

6. **Relation between trigonometric ratios**:

The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

i. $\text{cosec}\theta = \frac{1}{\sin\theta}$

ii. $sec\theta = \frac{1}{\cos\theta}$

iii. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

iv. $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

7. **Values of Trigonometric ratios** of some specific angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- The value of sin *A* or cos A never exceeds 1, whereas the value of sec *A* or cosec *A* is always greater than 1 or equal to 1.

- The value of sinθ increases from 0 to 1 when θ increases from 0^o to 90^o.
- The value of cosθ decreases from 1 to 0 when θ increases from 0^o to 90^o.
- If one of the sides and any other parts like either an acute angle or any side of a right triangle are known, the remaining sides and angles of the triangle can be obtained using trigonometric ratios.

8. **Trigonometric ratios of complementary angles**:

Two angles are said to complementary angles if their sum is equal to 90^o. Based on this relation, the trigonometric ratios of complementary angles are given as follows:

i. $\sin(90^o - A) = \cos A$

ii. $\cos(90^o - A) = \sin A$

iii. $\tan(90^o - A) = \cot A$

iv. $\cot(90^o - A) = \tan A$

v. $\sec(90^o - A) = \text{cosec } A$

vi. $\text{cosec}(90^o - A) = \sec A$

Note: $\tan 0^o = 0 = \cot 90^o$, $\sec 0^o = 1 = \text{cosec } 90^o$, $\sec 90^o$, $\text{cosec } 0^o$, $\tan 90^o$ and $\cot 0^o$ are not defined.

9. Definition of Trigonometric Identity

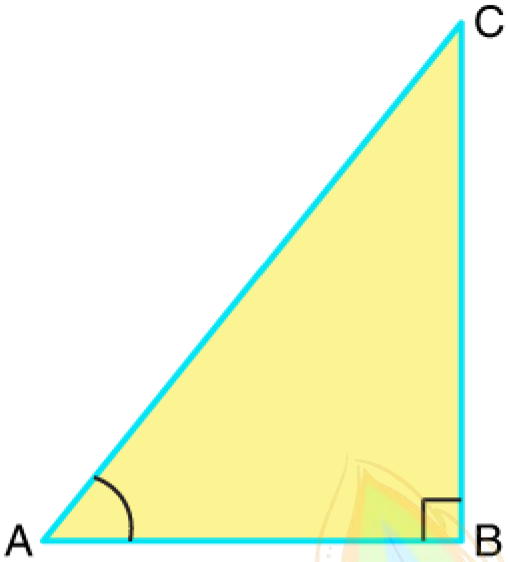
An equation involving trigonometric ratios of an angle, say θ, is termed as a **trigonometric identity** if it is satisfied by all values of θ.

10. Basic trigonometric identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta;\ 0 \le \theta < 90^o$
- $1 + \cot^2\theta = \text{cosec}^2\theta;\ 0 \le \theta < 90^o$

11. Opposite & Adjacent Sides in a Right Angled Triangle

In the ΔABC right-angled at B, BC is the side opposite to ∠A, AC is the hypotenuse and AB is the side adjacent to ∠A.



12. Trigonometric Ratios

For the right ΔABC, right-angled at ∠B, the trigonometric ratios of the ∠A are as follows:

sin A = opposite side/hypotenuse = BC/AC

cos A = adjacent side/hypotenuse = AB/AC

tan A = opposite side/adjacent side = BC/AB

cosec A = hypotenuse/opposite side = AC/BC

sec A = hypotenuse/adjacent side = AC/AB

cot A = adjacent side/opposite side = AB/BC

13. Visualization of Trigonometric Ratios Using a Unit Circle

Draw a circle of the unit radius with the origin as the centre. Consider a line segment OP joining a point P on the circle to the centre which makes an angle θ with the x-axis. Draw a perpendicular from P to the x-axis to cut it at Q.

Sinθ = PQ/OP=PQ/1=PQ

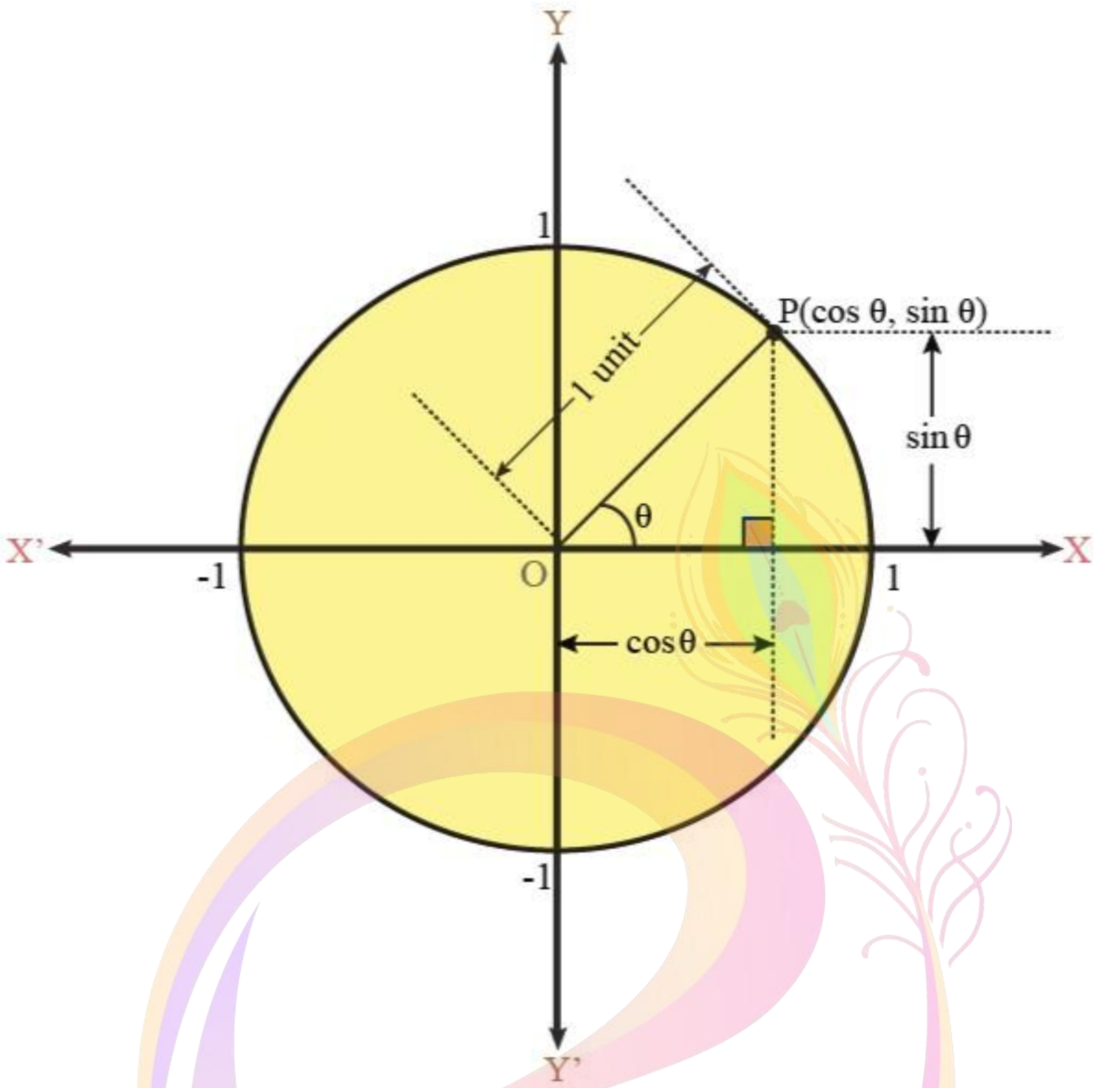
cosθ = OQ/OP=OQ/1=OQ

tanθ = PQ/OQ=sinθ/cosθ

cosecθ = OP/PQ=1/PQ

secθ = OP/OQ=1/OQ

cotθ = OQ/PQ=cosθ/sinθ



14. Relation between Trigonometric Ratios

cosec θ = 1/sin θ

sec θ = 1/cos θ

tan θ = sin θ/cos θ

cot θ = cos θ/sin θ = 1/tan θ

15. Range of Trigonometric Ratios from 0 to 90 degrees

For $0^\circ \leq \theta \leq 90^\circ$,

- $0 \leq \sin\theta \leq 1$
- $0 \leq \cos\theta \leq 1$
- $0 \leq \tan\theta < \infty$
- $1 \leq \sec\theta < \infty$
- $0 \leq \cot\theta < \infty$
- $1 \leq \text{cosec}\theta < \infty$

tanθ and secθ are not defined at 90°.

cotθ and cosecθ are not defined at 0°.

16. Variation of trigonometric ratios from 0 to 90 degrees

As θ increases from 0° to 90°

sin θ increases from 0 to 1

cos θ decreases from 1 to 0

tan θ increases from 0 to ∞

cosec θ decreases from ∞ to 1

sec θ increases from 1 to ∞

cot θ decreases from ∞ to 0

17. Standard values of Trigonometric ratios

∠A	0°	30°	45°	60°	90°
sin A	0	1/2	1/√2	√3/2	1
cos A	1	√3/2	1/√2	1/2	0
tan A	0	1/√3	1	√3	not defined
cosec A	not defined	2	√2	2/√3	1
sec A	1	2/√3	√2	2	not defined
cot A	not defined	√3	1	1/√3	0

18. Complementary Trigonometric ratios

In Mathematics, the complementary angles are the set of two angles such that their sum is equal to 90°. For example, 30° and 60° are complementary to each other as their sum is equal to 90°. In this article, let us discuss in detail about the complementary angles and the trigonometric ratios of complementary angles with examples in a detailed way.

If θ is an acute angle, its complementary angle is 90° − θ. The following relations hold true for trigonometric ratios of complementary angles.

sin (90° − θ) = cos θ

cos (90° − θ) = sin θ

tan (90° − θ) = cot θ

cot (90° − θ) = tan θ

cosec (90° − θ) = sec θ

sec (90° − θ) = cosec θ