NOTES

– NOTES WITH MIND MAPS –

MATHEMATICS

(TRIANGLES)

10TH

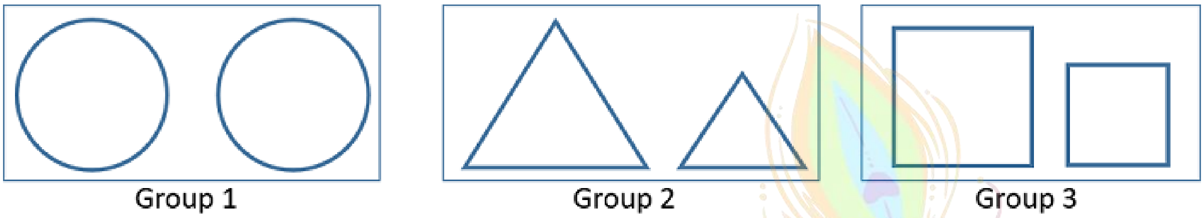
Triangles

1. **Congruent figures:**

   Two geometrical figures are called **congruent** if they superpose exactly on each other, that is, they are of the same shape and size.

2. **Similar figures:**

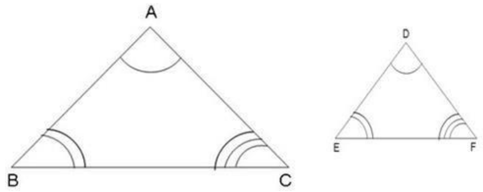
   Two figures are **similar**, if they are of the same shape but not necessarily of the same size.



3. All congruent figures are similar but the similar figures need not to be congruent.
4. Two **polygons** having the same number of sides are **similar** if
   i. their corresponding angles are equal and
   ii. their corresponding sides are in the same ratio (or proportion).

   Note: Same ratio of the corresponding sides means the **scale factor** for the polygons.
5. Important facts related to similar figures are:
   i. All circles are similar.
   ii. All squares are similar.
   iii. All equilateral triangles are similar.
   iv. The ratio of any two corresponding sides in two equiangular triangles is always same.
6. Two **triangles are similar** (~) if
   i. Their corresponding angles are equal.
   ii. Their corresponding sides are in same ratio.
7. If the angles in two triangles are:
   i. Different, the triangles are neither similar nor congruent.
   ii. Same, the triangles are similar.
   iii. Same and the corresponding sides are of the same size, the triangles arecongruent.

In the given figure, $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, which means triangles ABC and DEF are similar which is represented by $\Delta ABC \sim \Delta DEF$



8. If $\Delta ABC \sim \Delta PQR$, then
   i. $\Delta A = \Delta P$
   ii. $\Delta B = \Delta Q$
   iii. $\Delta C = \Delta R$
   iv. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

9. **Equiangular triangles:**

   Two triangles are **equiangular** if their corresponding angles are equal. The ratio of any two corresponding sides in such triangles is always the same.

10. **Basic Proportionality Theorem** (Thales Theorem):

    If a line is drawn parallel to one side of a triangle to intersect other two sides in distinct points, the other two sides are divided in the same ratio.

11. **Converse of BPT:**

    If a line divides any two sides of a triangle in the same ratio then the line is parallel to the third side.

12. A line drawn through the mid-point of one side of a triangle which is parallel to another side bisects the third side. In other words, the line joining the mid-points of any two sides of a triangle is parallel to the third side.

13. **AAA (Angle-Angle-Angle) similarity criterion:**

    If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

14. **AA (Angle-Angle) similarity criterion:**

    If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal.

    Thus, **AAA similarity criterion** changes to **AA similarity criterion** which can be stated as follows:

    If two angles of one triangle are respectively equal to two angles of other triangle, then the two triangles are similar.

15. **Converse of *AAA* similarity criterion:**

    If two triangles are similar, then their corresponding angles are equal.

16. **SSS (Side-Side-Side) similarity criterion:**

    If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

17. **Converse of SSS similarity criterion:**

    If two triangles are similar, then their corresponding sides are in constant proportion.

18. **SAS (Side-Angle-Side) similarity criterion:**

    If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

19. **Converse of SAS similarity criterion:**

    If two triangles are similar, then one of the angles of one triangle is equal to the corresponding angle of the other triangle and the sides including these angles are in constant proportion.

20. **RHS (Right angle-Hypotenuse-Side) criterion:**

    If in two right triangles, hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of another triangle, then the two triangles are similar. This criteria is referred as the RHS similarity criterion

21. **Pythagoras Theorem:**

    In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

    Thus, in triangle ABC right angled at B, $AB^2 + BC^2 = AC^2$

22. **Converse of Pythagoras Theorem:**

    If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

23. The **ratio of the areas of two similar triangles** is equal to the square of the ratio of their corresponding sides.

    Thus, if $\Delta ABC \sim \Delta PQR$, then $\frac{ar\,\Delta ABC}{ar\,\Delta PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

    Also, the ration of the areas of two similar triangles is equal to the ration of the squares of the corresponding medians.

24. Some important results of similarity are:

    In an equilateral or an isosceles triangle, the altitude divides the base into two equal parts.

    If a perpendicular is drawn from the vertex of the right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

    The area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonal.

    Sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

In an equilateral triangle, three times the square of one side is equal to four times the square of one of its altitudes.

25. **Triangle**

A triangle can be defined as a polygon which has three angles and three sides. The interior angles of a triangle sum up to 180 degrees and the exterior angles sum up to 360 degrees. Depending upon the angle and its length, a triangle can be categorized in the following types-

- Scalene Triangle – All the three sides of the triangle are of different measure
- Isosceles Triangle – Any two sides of the triangle are of equal length
- Equilateral Triangle – All the three sides of a triangle are equal and each angle measures 60 degrees
- Acute angled Triangle – All the angles are smaller than 90 degrees
- Right angle Triangle – Anyone of the three angles is equal to 90 degrees
- Obtuse-angled Triangle – One of the angles is greater than 90 degrees

26. Similarity Criteria of Triangles

To find whether the given two triangles are similar or not, it has four criteria. They are:

Side-Side- Side (SSS) Similarity Criterion – When the corresponding sides of any two triangles are in the same ratio, then their corresponding angles will be equal and the triangle will be considered as similar triangles.

Angle Angle Angle (AAA) Similarity Criterion – When the corresponding angles of any two triangles are equal, then their corresponding side will be in the same ratio and the triangles are considered to be similar.

Angle-Angle (AA) Similarity Criterion – When two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are considered as similar.

Side-Angle-Side (SAS) Similarity Criterion – When one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are said to be similar.

27. Proof of Pythagoras Theorem

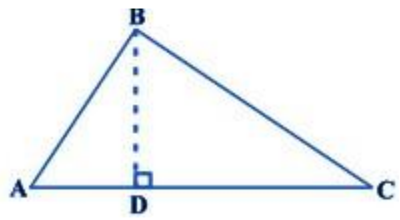
Statement: As per Pythagoras theorem, "In a right-angled triangle, the sum of squares of two sides of a right triangle is equal to the square of the hypotenuse of the triangle."

Proof –

Consider the right triangle, right-angled at B.

Construction-

Draw BD ⊥ AC



Now, △ADB ~ △ABC

So, AD/AB = AB/AC

or AD. AC = AB^2..................... (i)

Also, △BDC ~ △ ABC

So, CD/BC = BC/AC

or, CD. AC = BC^2..................... (ii)

Adding (i) and (ii),

AD. AC + CD. AC = $AB^2 + BC^2$

AC (AD + DC) = $AB^2 + BC^2$

AC (AC) = $AB^2 + BC^2$

⇒ $AC^2 = AB^2 + BC^2$

Hence, proved.

28. Problems Related to Triangles

A girl having a height of 90 cm is walking away from a lamp-post's base at a speed of 1.2 m/s. Calculate the length of that girl's shadow after 4 seconds if the lamp is 3.6 m above the ground.

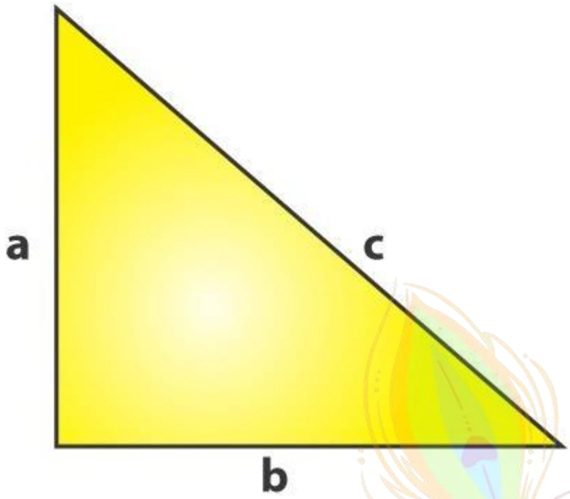
S and T are points on sides PR and QR of triangle PQR such that angle P = angle RTS. Now, prove that triangle RPQ and triangle RTS are similar.

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD

at F. Show that triangles ABE and CFB are similar.

29. Pythagoras Theorem Statement

Pythagoras theorem states that "In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides". The sides of this triangle have been named as Perpendicular, Base and Hypotenuse. Here, the hypotenuse is the longest side, as it is opposite to the angle 90°. The sides of a right triangle (say a, b and c) which have positive integer values, when squared, are put into an equation, also called a Pythagorean triple.



History

The theorem is named after a greek Mathematician called Pythagoras.

Pythagoras Theorem Formula

Consider the triangle given above:

Where “a” is the perpendicular,

“b” is the base,

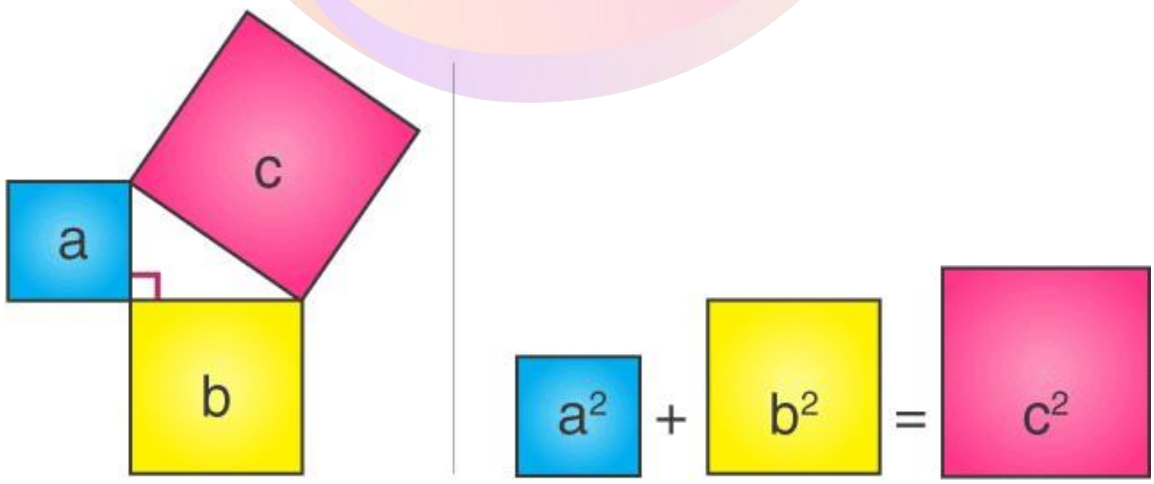
“c” is the hypotenuse.

According to the definition, the Pythagoras Theorem formula is given as:

$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$

$c^2 = a^2 + b^2$

The side opposite to the right angle (90°) is the longest side (known as Hypotenuse) because the side opposite to the greatest angle is the longest.



Consider three squares of sides a, b, c mounted on the three sides of a triangle having the same sides as shown.

By Pythagoras Theorem –

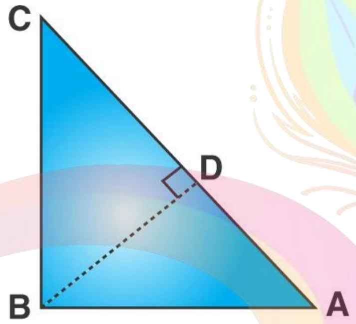
Area of square "a" + Area of square "b" = Area of square "c"

Pythagoras Theorem Proof

Given: A right-angled triangle ABC, right-angled at B.

To Prove- $AC^2 = AB^2 + BC^2$

Construction: Draw a perpendicular BD meeting AC at D.



Proof:

We know, △ADB ~ △ABC

Therefore,

$$\frac{AD}{AB} = \frac{AB}{AC}$$

(corresponding sides of similar triangles)

Or, $AB^2 = AD \times AC$...(1)

Also, △BDC ~△ABC

Therefore,

$$\frac{CD}{BC} = \frac{BC}{AC}$$

(corresponding sides of similar triangles)

Or, $BC^2 = CD \times AC$... (2)

Adding the equations (1) and (2) we get,

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$AB^2 + BC^2 = AC (AD + CD)$

Since, AD + CD = AC

Therefore, $AC^2 = AB^2 + BC^2$

Hence, the Pythagorean theorem is proved.